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БИР КАЛЫПТУУ ЛИНДЕЛЁВДУК МЕЙКИНДИКТЕР

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РАВНОМЕРНО ЛИНДЕЛЁФОВЫ ПРОСТРАНСТВА

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Бул илимий макалада бир калыптуу линделёвдук мейкиндик түшүнүгү киргизилген жана изилденген. Бир калыптуу линделёвдук мейкиндик линделёвдук мейкиндиктин бир калыптуу аналогу болуп саналат. Ар бир сепарабелдүү метризацияланган мейкиндик бир калыптуу линделёвдук мейкиндик экендиги көрсөтүлгөн. Бир калыптуу мейкиндиктердин каралып жаткан классы менен компактуу жана күчтүү бир калыптуу B -паракомпактуу мейкиндиктердин классынын арасындагы байланыштар тургузулуп, бул класс компактуу жана күчтүү бир калыптуу B - паракомпактуу мейкиндиктердин класстарынын арасында жайгашкандыгы тастыкталган. Алынган натыйжалардын өзгөчө маанилүүсү болуп А. Борубаевдин «кайсыл бир калыптуу мейкиндиктер каалагандай чектүү аддитивдүү ачык ω жабдуу үчүн кандайдыр бир сепарабелдуу метризацияланган мейкиндикке карата ω -чагылдырууга ээ болот?» деп коюлган маселесинин чечилиши саналат. Ошону менен эле бирге бир калыптуу линделёвдук мейкиндик бир калыптуу туюк чагылдырууда образ тарабына, ал эми бир калыптуу жеткилен чагылдырууда прообраз тарабына сакталышы далилденген.

***Негизги сөздөр:** бир калыптуу мейкиндик, бир калыптуу линделёвдук мейкиндик, чектүү аддитивдүү жабдуу.*

В настоящей статье вводится и изучается равномерно линделёфовы пространства. Равномерно линделёфовы пространства являются равномерными аналогами линделёфовых пространств. В частности показана, что каждое сепарабельное метризуемое пространство является равномерно линделёфовыми. Установлены связи рассматриваемого класса равномерных пространств между классами компактных и сильно равномерно B - паракомпактных пространств, выяснено, что этот класс находится между классами компактных и сильно равномерно B - паракомпактных пространств. Наиболее важным результатом является решение проблемы А. Борубаева: «каковы те равномерные пространства которые обладают ω - отображения на некоторое сепарабельно метризуемое равномерное пространство для любого конечно аддитивного открытого покрытия ω ?». Доказана, что при равномерно замкнутых отображениях равномерная линделёфовость сохраняется в сторону образа, а при равномерно совершенных отображениях в сторону прообраза.

***Ключевые слова:** равномерное пространство, равномерно линделёфово пространство, конечно аддитивное покрытие.*

In this article we introduce and study uniformly Lindelof spaces. Uniformly Lindelof spaces are uniform analogies of Lindelof spaces. In particular, it is shown that every separable metrizable space is uniformly Lindelof. The connection between the classes of compact and strongly uniformly B - paracompact spaces is established, that is, it is clarified that this class is strictly between the classes of compact and strongly uniformly B - paracompact spaces. The main result of the work is the solution of the problem posed by A. Borubaev: "what are uniform spaces which have uniformly continuous ω -mapping on separable metric uniform space for any finitely additive open covering ω ?". It is proved that for uniformly open and also uniformly closed maps, the uniform Lindelofness is preserved in the direction of the image, and for uniform perfect mapping to the inverse image.

***Key words:** uniform space, uniformly Lindelof space, finitely additive covering.*

The class of Lindelof spaces is important in general topology. There are different approaches to the definition of uniformly Lindelof space. Everywhere in this article, topological spaces are assumed to be Tychonoff, uniform spaces to be Hausdorff and mappings to be uniformly continuous. For the uniformity U by τ_U we denote the topology generated by the uniformity. For the covering α of the set X and $x \in X$, $H \subset X$ we have: $\alpha(x) = \cup\{A \in \alpha : A \ni x\}$, $\alpha(H) = \cup\{A \in \alpha : A \cap H \neq \varnothing\}$. For the cover α and β of the set X, that symbols $\alpha \succ \beta$, and $\alpha * \succ \beta$ means respectively that the cover α is a refinement of the cover β, and the cover α is a strongly star refinement of the cover β, i.e. $\{\alpha(A); A \in \alpha\} \succ \beta$. For the covering α of subsets of the set X and a finitely cardinal number $\aleph_0$ let $\alpha_{\aleph_0} = \{\cup\alpha_0 : \alpha_0 \subset \alpha, |\alpha_0| \leq \aleph_0\}$. The covering α will be $\aleph_0$-additive or finitely-additive if $\alpha_{\aleph_0} \succ \alpha$.

We can now formulate the definitions of uniformly Lindelof space belonging to Isbell [1], Aparina [2] and Borubaev [3].

Definition 1. [1] A uniform space (X, U) is said to be uniformly I-Lindelof, if every uniformly covering admits a countable uniformly covering refinement.

Definition 2. [2] A uniform space (X,U) is said to be uniformly A -Lindelof, if for every open covering α there exist a countable uniformly covering $\beta=\{B_n;n\in N\}$ and $\gamma\in U$ such that $\beta\succ\alpha_{\aleph_0}$ and $\gamma(\overline{B}_n)\subset B_{n+1}$ for $n\in N$.

In the work [2] it is proved that any uniformly A -Lindelof space is complete. From this follows that any separable metrizable uniform space, by a non complete separable metric, uniform spaces are not uniformly A - Lindelof spaces.

The uniform space (X,U) is said to be uniformly B -paracompact [3], if for each finitely additive open cover γ of (X,U) there exists such sequence uniform covering $\{\alpha_i : i\in N\}\subset U$, that following condition is realized:

(UB). For each point $x\in X$ there exist such number $i\in N$ and $\Gamma\in\gamma$ that $\alpha_i(x)\subset\Gamma$.

Definition 3. [3], [5]. A uniform space (X,U) is said to be uniformly B -Lindelof, if it is uniformly B -paracompact and I-Lindelof.

Definition 4. A uniform space (X,U) is said to be uniformly Lindelof, if for each finitely additive open covering γ of (X,U) there exists such a sequence of countable uniform covering $\{\alpha_n : n\in N\}\subset U$, with property (UB).

Proposition 1. Any separable metrizable uniform apace (X,U) is uniformly Lindelof.

Proof. Let (X,U) be separable metrizable uniform apace. Then the uniform space (X,U) has been countable base $B=\{\alpha_n : n\in N\}$ consisting is countable coverings. We prove that the any finitely-additive open cover γ of (X,U), the a sequence countable covering $\{\alpha_n : n\in N\}\subset U$, realizing the condition (UB). Let $x\in X$ be arbitrary point. Then there exist $\Gamma\in\gamma$, such that $\Gamma\ni x$. Due to definition of topological space (X,τ_U) exist such uniform covering $\alpha\in U$, that $\alpha(x)\subset\Gamma$. Then there exist such number $n\in N$, that $\alpha_n\succ\alpha$. Its clear that $\alpha_n(x)\subset\alpha(x)\subset\Gamma$. Thus, the uniform space (X,U) is uniformly Lindelof.

Proposition 2. If (X,U) is uniformly Lindelof space then the topological space (X,τ_U) is Lindelof. Conversely, if (X,τ) is Lindelof then the uniform space (X,U_X) is uniformly Lindelof, where U_X is a universally uniformities of the space (X,τ).

Proof. Let γ be any arbitrary finitely additive open covering of the space (X,τ_U). Then exists a normal sequence of countable uniformly coverings $\{\alpha_n : n\in N\}\subset U$, realizing the condition (UB). Then, due to separable metrizational lemma, there exists such separable pseudometric d on X , that $\alpha_{n+1}(x)\subset\{y: d(x,y)<\frac{1}{2^{n+1}}\}\subset\alpha_n(x)$, for all $x\in X$, $n\in N$. The family $<\gamma>=\{<\Gamma>:\Gamma\in\gamma\}$ is an open covering of the separable pseudometric space (X,d). From the fact that every separable metrizable space is a Lindelof that it follows that there is a countable covering σ of the space (X,τ_ρ) refined in an open covering γ. Since $\tau_\rho\subset\tau_U$, then σ is countable open covering of the space (X,τ_U). Therefore, (X,τ_U) is Lindelof. Conversely, let (X,τ) be a Lindelof space. Then the set of all open coverings forms base the universal uniformity U_X of the space (X,τ). Then, it is easy to derive from separable metrizational lemma, that (X,U_X) is uniformly Lindelof space.

Every uniformly B -Lindelof space is uniformly Lindelof.

Remember [3] a continuous mapping $f : X \to Y$ of a topological space X to a topological space Y is said to be a ω - mapping, if for each point $y \in Y$ there exist such neighborhood $O \ni y$ and $W \in \omega$, that $f^{-1}(O) \subset W$.

Theorem 1. For the uniform space (X, U) the following conditions are equivalent:

1. The uniform space (X, U) is uniformly Lindelof.
2. For each finitely additive open cover ω of (X, U) there exists a uniformly continuous ω -mapping f of the uniform space (X, U) onto a separable metrizable uniform space (Y, V).

Proof. $1 \Rightarrow 2$. Let (X, U) be a uniformly Lindelof space and ω be a finitely additive open covering of the space (X, U). Then for a covering ω there exists a normal sequence of the countable coverings $\{\alpha_n\}$, realizing the property (UB). For $\{\alpha_n\}$, there exists such separable pseudometric d on X, that $\alpha_{n+1}(x) \subset \{y : d(x, y) < \frac{1}{2^{n+1}}\} \subset \alpha_n(x)$, for all $x \in X$ and for all $n \in N$. Introduce the relation of equivalence: $x \sim y$ if and only if $d(x, y) = 0$, for any $x, y \in X$. Let Y be the factor set of the set X relative to the equivalence relation "$\sim$" and $f : X \to Y$ is natural projection. Denote $\rho(y_1, y_2) = d(f^{-1}y_1, f^{-1}y_2)$ for all $y_1, y_2 \in Y$. It is easy to check that ρ is a separable metric. Let V be a uniformity on Y induced from the separable metric ρ. The map $f : (X, U) \to (Y, V)$ is uniformly continuous. Let $y \in Y$ be an arbitrary point and x be any point in $f^{-1}y$. Then there exist such number $n \in N$ and $W \in \omega$, that $\alpha_n(x) \subset W$. Denote $O_y = \{y \in Y : \rho(y, y) < \frac{1}{2^{n+2}}\}$. Then $f^{-1}O_y \subset \{x \in X : \rho(x, x) \le \frac{1}{2^{n+1}}\} \subset \alpha_n(x) \subset W$. Hence, f is a ω -mapping.

$2 \Rightarrow 1$. Let ω be an arbitrary finitely additive open covering of the space (X, U), and $f : (X, U) \to (Y_\omega, V_\omega)$ be a uniformly continuous ω -mapping of the uniform space (X, U) onto a separable metrizable uniform space (Y_ω, V_ω). We show that the uniform space (X, U) is a uniformly Lindelof space. Then exists a base consisting of countable coverings $\{\beta_n\}$ of the space (Y_ω, V_ω). Denote $\{\alpha_n\}$, where $\alpha_n = f^{-1}\beta_n$. Obviously, $\{\alpha_n\}$ is a family of the countable uniform coverings of the space (X, U). We prove that for every point $x \in X$ exists number $n \in N$ and $W \in \omega$ such, that $\alpha_n(x) \subset W$. Let $x \in X$ is an arbitrary point. Then for point $y \in Y$, $y = f(x)$ there exist $O_y \ni y$ и $W \in \omega$ such that $f^{-1}O_y \subset W$. Hence, for point $x \in f^{-1}y \subset f^{-1}O_y$ exists such number $n \in N$, that $\alpha_n(x) \subset f^{-1}O_y \subset W$. Thus, the uniform space (X, U) is uniformly Lindelof.

Proposition 3. Each closed subspace (H, U_H) of a uniformly Lindelof space (X, U) is uniformly Lindelof.

Proof. Let α_H be an arbitrary finitely additive open covering of the subspace (H, U_H). Then there exist such open family η of (X, U), that $\alpha_H = \eta \wedge \{H\}$. Denote $\alpha = \{\eta, X \setminus H\}$. It‘s clear that α is an open covering of the subspace (X, U). Since the latter is uniformly Lindelof, then for α exists a sequence countable uniform covering $\{\gamma_n\}$, with properties (UB). Denote $\{\gamma_n^H\}$, $\gamma_n^H = \gamma_n \wedge \{H\}$. Obviously, $\{\gamma_n^H\}$ is a sequence countable uniform coverings of the space (H, U_H). It is easy to see that, for each point $x \in H$ exists such number $n \in N$ and $A_H \in \alpha_H$, that $\gamma_n^H(x) = \gamma_n(x) \cap H \subset A \cap H = A_H$. Hence, (H, U_H) is a uniformly Lindelof space.

Remember [4] that the uniform space (X,U) is said to be uniformly locally Lindelof, if there exists such uniform covering $\alpha \in U$ that the closures of all its elements are Lindelof.

Proposition 4. Any uniformly Lindelof space is uniformly locally Lindelof.

Proof implies from proposition 2 and 3.

Corollary 1. Any compact uniform space is uniformly Lindelof.

The uniform space (X,U) is called a strongly uniformly B-paracompact, if it is a uniformly B-paracompact and the topological space (X,τ_U) are strongly paracompact.

Proposition 5. If (X,U) is strongly uniformly B-paracompact space then the topological space (X,τ_U) is strongly paracompact. Conversely, if (X,τ) is strongly paracompact then the uniform space (X,U_X) is strongly uniformly B-paracompact, where U_X is a universally uniformities of the space (X,τ).

Proof. Let ω be any arbitrary finitely additive open covering of the topological space (X,τ_U) and $f:(X,\tau_U)\to(Y,\tau_V)$ is a ω-map of the space (X,τ_U) onto the space (Y,τ_V). Then there exists an open cover η in Y_ω such that $f^{-1}\eta \succ \lambda$. If μ is a star finitely open refinement of the cover η then the cover $f^{-1}\mu$ is open star finitely cover and is a refinement of the cover ω. Hence, (X,τ_U) is strongly paracompact. Conversely, let (X,τ) be a strongly paracompact space. Obviously, set of all open coverings forms base the universal uniformity U_X of the space (X,τ). Then, it is easy to derive from separable metrizational lemma, that (X,U_X) is strongly uniformly B - paracompact space.

Theorem 2. For the uniform space (X,U) the following conditions are equivalent:

1. The uniform space (X,U) is strongly uniformly B-paracompact.
2. For each finitely additive open cover ω of (X,U) there exists a uniformly continuous ω-mapping f of the uniform space (X,U) onto a strongly paracompact metrizable uniform space (Y_ω,V_ω).

Proof. $1\Rightarrow 2$. Let (X,U) be a uniformly strongly B-paracompact space and ω be a finitely additive open covering of the space (X,U). Then for a covering ω there exists a normal sequence of the uniform coverings $\{\alpha_n\}$, with property (UB). For $\{\alpha_n\}$, there exists such pseudometric d on X, that $\alpha_{n+1}(x)\subset\{y:d(x,y)<\frac{1}{2^{n+1}}\}\subset\alpha_n(x)$, $x\in X$, $n\in N$. For any $x,y\in X$ let $x\sim y$ if and only if $d(x,y)=0$. Let Y_ω be the factor set of the set X relative to the equivalence relation "$\sim$" and $f:X\to Y$ is natural map. We put $\rho(y_1,y_2)=d(f^{-1}y_1,f^{-1}y_2)$ for all $y_1,y_2\in Y$. Then $\rho(y_1,y_2)$ is a metric on the set Y_ω. Let V_ω be a uniformity on Y_ω induced from the metric ρ. Then by inclusion $\alpha_{n+1}(x)\subset\{y:d(x,y)<\frac{1}{2^{n+1}}\}\subset\alpha_n(x)$, $x\in X$, $n\in N$ we have the map $f:(X,U)\to(Y_\omega,V_\omega)$ is uniformly continuous. We show that f is a ω-mapping. Let $y\in Y$ be an arbitrary point and $x\in f^{-1}y$. Then there exist such number $n\in N$ and $W\in\omega$, that $\beta_n(x)\subset W$. Denote $O_y=\{y\in Y:\rho(y,y)<\frac{1}{2^{n+2}}\}$. Then $f^{-1}O_y\subset\{x\in X:\rho(x,x)\le\frac{1}{2^{n+1}}\}\subset\beta_n(x)\subset W$. Consequently, f is a ω-mapping.

$2\Rightarrow 1$. Let ω be an arbitrary a finitely additive open covering of (X,U), and $f:(X,U)\to(Y_\omega,V_\omega)$ be a uniformly continuous ω-mapping of the uniform space (X,U) onto a metrizable uniform space (Y,V). For every point $y\in Y_\omega$ there exists a neighborhood $O_y\ni y$, the inverse image of which lies in some element of the ω . Then exists a sequence of countable uniform coverings $\{\beta_n\}$ of (Y_ω, V_ω), for which property (UB) holds. Denote $\{\alpha_n\}$, where $\alpha_n = f^{-1}\beta_n$. Obviously, $\{\alpha_n\}$ is a sequence of the countable uniform coverings of the space (X, U). Then it is easy to see that the sequence $\{\alpha_n\}$, $n \in N$ has property (UB). Now we prove that the open covering ω exist such star finitely open covering γ, that $\gamma \succ \omega$. Denote $\lambda = \{O_y : y \in Y_\omega\}$, then there exists an star finitely open cover η in Y_ω such that $\eta \succ \lambda$. Hence, $f^{-1}\eta \succ \omega$. If η is a star finitely open cover then cover $f^{-1}\eta$ is an open star finitely cover of the space X. Thus, the uniform space (X, U) is strongly uniformly B-paracompact.

Theorem 3. Let (X, U) be a uniform space and cX certain compact Hausdorff extension of the space (X, τ_U). Then the following conditions are equivalent:

1. A uniform space (X, U) is uniformly Lindelof.
2. For each compact $K \subset cX \setminus X$ there exists a sequence countable uniform coverings $\{\alpha_n\}$, realizing the condition:

for each point $x \in X$ there exists such number $n \in N$, that $[\alpha_n(x)]_{cX} \cap K = \varnothing$.

Proof. $1 \Rightarrow 2$. Let (X, U) be a uniformly Lindelof space, cX be certain compact Hausdorff extension of the space (X, τ_U) and $K \subset cX \setminus X$ be an arbitrary compact set. Denote the set of all such open subsets of Γ compact cX, that $[\Gamma]_{cX} \cap K = \varnothing$ as γ. Let $\beta = \{\Gamma \cap X : \Gamma \in \gamma\}$. It is easy to see that, the covering β is a finitely additive open covering of the space (X, U). Then exists such a sequence countable uniform coverings $\{\alpha_n\}$, that for each point $x \in X$ exists number $n \in N$ such, that $\alpha_n(x) \subset \Gamma \cap X$ for $\Gamma \in \gamma$. Hence, $[\alpha_n(x)]_{cX} \cap K = \varnothing$. Further, in the finitely additive open covering β, an star finitely open covering α can be refined. Then for any $A \in \alpha$ exist such $B \in \beta$, that $A \subset B = \Gamma \cap X$. Thus, $[A]_{cX} \cap K = \varnothing$.

$2 \Rightarrow 1$. Let β be an arbitrary finitely additive open covering of the space (X, U). Than exists such family γ of open subsets of the certain compact Hausdorff extension cX, that $\beta = \{\Gamma \cap X : \Gamma \in \gamma\}$. The set $K = cX \setminus \cup\{\Gamma : \Gamma \in \gamma\}$ is a compact. By the condition of the theorem, for the compact $K \subset cX \setminus X$ exist such a sequence countable uniform covering $\{\alpha_n\}$, realizing the condition: for each point $x \in X$ there exists such number $n \in N$, that $[\alpha_n(x)]_{cX} \cap K = \varnothing$. Then there exist such $\{\Gamma_1, \Gamma_2, ..., \Gamma_m\} \subset \gamma$, that $[\alpha_n(x)]_{cX} \subset \bigcup_{n=1}^{m} \Gamma_n$. Consequently, $\alpha_n(x) \subset (\bigcup_{n=1}^{m} \Gamma_n) \cap X$. The covering β is a finitely-additive, therefore $(\bigcup_{n=1}^{m} \Gamma_n) \cap X \in \beta$.

Proposition 6. Every uniformly Lindelof space is a strongly uniformly B-paracompact.

Proof. Every uniformly Lindelof space is a uniformly B-paracompact and its topological space (X, τ_U) is a Lindelof. Hence, the uniform space (X, U) is a strongly uniformly B-paracompact.

Proposition 7. Any strongly uniformly B-paracompact space is uniformly B-paracompact.

A uniformly continuous mapping $f : (X, U) \to (Y, V)$ of a uniform space (X, U) to a uniform space (Y, V) is said to be uniformly perfect, if it is both precompact and perfect.

Theorem 4. The uniformly perfect preimage of the uniformly Lindelof space is a uniformly Lindelof.

Proof. Let $f : (X, U) \to (Y, V)$ be a uniformly perfect mapping of a uniform space (X, U) onto a uniform space (Y, V) and ω be an arbitrary finitely additive open covering of (X, U). The covering $\{f^{-1}y : y \in Y\}$ is a refined in the covering ω. By virtue of closedeness of the mappings f, the covering $\beta = \{f^{\#}O : O \in \omega\}$ is finitely additive open covering of the uniform space (Y, V) where

$f^{\#}O = Y \setminus f(X \setminus O)$. Since the uniform space (Y,V) is a uniformly Lindelof, then there exists a normal sequence of the countable covering $\{\alpha_i\}$, realizing the condition: for every point $y \in Y$ such exists number $i \in N$ and $f^{\#}O \in \beta$, that $\beta_i(y) \subset f^{\#}O$. It is easy to see that the covering $f^{-1}\beta$ is refined by the coverings ω. Let $x \in X$ be an arbitrary point. For the point $y \in Y$, $y = f(x)$ exists such number $i \in N$ and $f^{\#}O \in \beta$, that $\beta_n(y) \subset f^{\#}O$. Then exist such $\alpha_n \in U$, that $f\alpha_n \succ \beta_n$. It is easy to check that $\alpha_n(x) \subset f^{-1}(f\alpha_n(y)) \subset f^{-1}\beta_n(y) \subset O$. Consequently, the uniform space (X,U) is a uniformly Lindelof.

Proposition 8. A product $(X,U)\times(Y,V)$ the uniformly Lindelof space (X,U) on the compact space (Y,V) is a uniformly Lindelof.

Proof. As is known, the projection $f:(X,U)\times(Y,V) \to (X,U)$ is a uniformly perfect mapping of a product $(X,U)\times(Y,V)$ onto uniformly space (X,U). Then the product $(X,U)\times(Y,V)$ is a uniformly Lindelof space according to theorem 4.

Remember [6] a uniformly continuous mapping $f:(X,U) \to (Y,V)$ of a uniform space (X,U) onto a uniform space (Y,V) is said to be uniformly open, if for each $\alpha \in U$ there exists such and $\beta \in V$ that $f(\alpha(x)) \supset \beta(f(x))$ for all $x \in X$. Also a uniformly continuous mapping $f:(X,U) \to (Y,V)$ of a uniform space (X,U) onto a uniform space (Y,V) is said to be uniformly closed, if for each $\alpha \in U$ there exists such $\beta \in V$ that $\alpha(f^{-1}(y)) \supset f^{-1}(\beta(y))$ for all $y \in Y$.

Proposition 9. The uniformly open image of the uniformly Lindelof space is a uniformly Lindelof.

Proof. Let $f:(X,U) \to (Y,V)$ be a uniformly open mapping of a uniform space (X,U) onto a uniform space (Y,V) and β be an arbitrary finitely additive open covering of (Y,V). Then $\alpha = f^{-1}\beta$ is a finitely additive open covering of the space (X,U). Since the space (X,U) is uniformly Lindelof, there exists a sequence $\{\alpha_n : n \in N\} \subset U$ countable uniform covering, that following condition is realized: for each point $x \in X$ there exist such number $n \in N$ and $A \in \alpha$ that $\alpha_n(x) \subset A$. In view of the uniform openness of the mapping f for $\alpha_n \in U$ exist $\beta_n \in V$ such that $f(\alpha_n(x)) \supset \beta_n(f(x))$ for all $x \in X$. Hence, for each point $y \in Y$ exists such $n \in N$ and $B \in \beta$, that $\beta_n(y) \subset B$. Therefore, the uniform space (X,U) is a uniformly Lindelof.

Proposition 10. The uniformly closed image of the uniformly Lindelof space is a uniformly Lindelof.

Proposition 11. For ω -mappings the uniform Lindelofness is preserved in the direction of the inverse image.

Let X be the subspace of the space l_1 consisting of all sequences $\{x_n\}$ of real numbers such that $0 \le x_n \le 1$ for all $n \in N$ and $x_n = 0$ for all $n \in N$, except perhaps for a single value. Then $\rho(x,y) = \sum_{n=1}^{\infty} |x_n - y_n|$, $x \in X$, $y \in Y$, $x = \{x_n\}$, $y = \{y_n\}$ is a metric on X. The space X is not complete, hence, not uniformly A -Lindelof. However, it is separable metrizable, hence uniformly Lindelof.

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