Панков П.С., Кененбаева Г.М.

«КӨПЧӨ» ЭФФЕКТИСИ БОЮНЧА ИЛИМИЙ БОЖОМОЛ ЖАНА БАШКА МАТЕМАТИКАДАГЫ ЭФФЕКТЕР

Панков П.С., Кененбаева Г.М.

ГИПОТЕЗА ОБ ЭФФЕКТЕ «МНОЖЕСТВЕННОСТИ» И ДРУГИЕ ЭФФЕКТЫ В МАТЕМАТИКЕ

Pavel S. Pankov, Gulai M. Kenenbaeva

HYPOTHESIS ON EFFECT OF «NUMEROSITY» AND OTHER EFFECTS IN MATHEMATICS

УДК: 517

Көп бөлүккө ээ болгон системада кубулуштар пайда болуу «көпчө» эффектиси деп аталган. Байыркы белгилүү синергетикалык процессти көрсөтүүчү, кокустан айырмалуу теңдемелер системасы курулган. Бул негизде диссипациялык системанын жаңы аныктамасы сунушталган. Макалада «эффект» жана «кубулуш» түшүнүктөрүнө жалпы аныктамалар көрсөтүлгөн. Ошондой эле, сингулярдуу козголуу теориясында кээ бир кубулуш көрсөтүлгөн. Алардын ичинде сингулярдуу циклдин жана тереңдөөчү чек катмардын кубулуштары.

Негизги сөздөр: *«көпчө», эффект, кубулуш, аныктама, дифференциалдык теңдеме, айырмалуу теңдеме, сингулярдуу дүүлүгүш*

Возникновение явлений только для систем с большим количеством компонент названо эффектом «множественности». Построена система случайных разностных уравнений, отражающая давно известный синергетический процесс. На этой основе, предложено новое определение диссипативной системы. В статье представлены общие определения понятий «эффекта» и «явления». Также представлены некоторые явления в теории сингулярных возмущений. Среди них - явления сингулярного цикла и углубляющегося пограничного слоя.

Ключевые слова: *множественность, эффект, явление, определение, сингулярное возмущение, дифференциальное уравнение, разностное уравнение, синергетика.*

Appearance of phenomena in systems only with large number of components is said to be the effect of numerosity. A system of random difference equations is built to simulate the ancient popular synergetic process. On this base, a new definition of dissipative system is proposed. General notions of "effect" and "phenomenon" are presented in the paper. Some phenomena in the theory of singular perturbations are also presented. There are phenomena of singular cycle and deepening boundary layer among them.

Key words: *numerosity, effect, phenomenon, definition, differential equation, difference equation, singular perturbation, synergetic.*

1. Introduction

Discoveries of new "phenomena" and "effects" used to be sufficient steps in developing science but there were not definitions of these notions before our publication [1]. We give corresponding definitions and examples, propose methodic to search new phenomena. Necessary conditions of occurrence of "phenomena" in the theory of singularly perturbed differential equations established and some phenomena are found.

2. Definitions

Consider a mathematical statement (theorem) in general as an implication of conditions $A \Rightarrow B$, or, more concrete, if there is a general class X of objects x and $A \subset X$, $B \subset X$ then $A \subset B$ or $((x \in A) \Rightarrow (x \in B))$. To search "phenomena" and "effects" more systematically we have proposed

DEFINITION 1 [1]. To prove sufficiency of A for B one is to construct an example without both A and B. An (interesting, nonpresumable, single) way of violating B is said to be a *phenomenon*.

The notion "single" can be defined more exactly. Let X be a set and a measure *mes* can be introduced in it. Then a subset $P \subset X \backslash B$ is a phenomenon if $mes(P)=0$. In other words, if $x \in X$ then $x \in P$ "almost never".

DEFINITION 2. If P is a property (or some properties) of elements $x \in X$ having a property E such that a logical proof $(E \wedge C) \Rightarrow P$ (where C is any additional condition) is too complicated and the property P was discovered not by a logical way but by meeting paradoxes, by experiments in physics and chemistry or by computational experiments in mathematics then E is said to be an *effect*.

These definitions yield the following methodic. If some objects $x \in X$ with different but similar unexpected properties have the same property E then this property is considered to be an effect. Putting additional conditions on x new phenomena may be found in the class X.

3. Phenomenon of irgöö and hypothesis on numerosity

The common Kyrgyz word *irgöö* means: discrete optimization by means of synergetic, or "random vibration of balls of different sizes of same material in a wide symmetrical vessel yields migration of the biggest one to the center of their surface." This experimental fact is too difficult to be proven by any mathematical model but can be corroborated by numerical experiments with a system of difference equations.

Thus, we stated

HYPOTHESIS 1 [2]. For a large number of balls in a vessel, in a certain class of processes described by random difference equations, the probability of the event "the biggest ball is close to the center of surface of heap of balls" is 1 as time tends to infinity.

The cylinder of radius 1 is taken as a vessel. Let a (large) natural number n and (small) positive radiuses $r_1> r_2\geq \ldots \geq r_n$ be given.

DEFINITION3. If a set of n points $\{(x_k, y_k, z_k): k=1..n\} \subset R^3$ fulfills the conditions

1) $r_k\leq z_k$, $x_k^2+ y_k^2\leq (1-r_k)^2$ for all k in $1..n$ (all balls are in the vessel);

2) $(x_k -x_j)^2+ (y_k-y_j)^2+ (z_k-z_j)^2\geq (r_j+ r_k)^2$ for all $k\neq j$ in $1..n$ (the balls do not overlap) then such set is said to be *admissible.*

DEFINITION 4. A (short) vector $\{u,v,w\}$ $(w<0)$ is said to be *admissible* for a given admissible set of points and a number k in $1..n$ if the set obtained by means of changing the k-th point to the point $(x_k+u, y_k+v, z_k+ w)$ isadmissible too. Such passing from one set of points to a new set of points is said to be an *admissible* shift.

ALGORITHM 1 (of approximate calculations).For any initial admissible set of points repeat the following steps:

1) shift all points up with a (short) vector;

2) while it is possible, in the obtained admissible set of points execute random admissible shifts.

The adjusted

HYPOTHESIS 2. With the probability 1, there exists such number M that after M steps there will be $x_1^2+ y_1^2\leq r_1^2$ and there will not be other points over this point.

To verify this hypothesis a program was written in *pascal* for $n=50$ and $r_k=0.3- 0.01k$, $k=1..19$; $r_k=0.1$, $k=20..50$.

Some runs of this program gave similar results corroborating the Hypothesis 2 with $M < 100$.

The phenomenon of Benard convection cells (1900) is mentioned in literature as the first example of synergetic process in a dissipative system (we did not find its implementations on computer). As the word *irgöö* existed in the Kyrgyz language some hundred years, this is the earlier example of synergetic process.

DEFINITION 3. If an open dynamicalsystem has “sufficiently many” possiblestates and such transitions between them that entropy of incoming energy is “sufficiently less” than entropy of outgoing energy then such system is said to be a *dissipative* one. Thus, the inner entropy decreases and it is equivalent to increasing of self-ordering.

REMARK. We did not find definitions of *entropy of energy*in literature.

DEFINITION 4. If any phenomenon (including those mentioned in Definition 3) arises for large number of components of system only then such phenomenon is said to be a consequence of *effect of numerosity*.

4. Phenomena in singularly perturbed differential equations

Let T and Y be topological spaces,εbe a small positive parameter, $\varepsilon_0$be an upper boundary for it and $\{y_\varepsilon(t):T\to Y\}$ be a family of continuous functions.

The phenomenon of boundary layer in physics of liquids and gases (zero-velocity of the layer of liquid or gas contiguous to a solid (Prandtl)); the theory of singular perturbations had been developed on this base.

This phenomenon is well-known but we did not find any strict definition for it. For $T\subset R$ and $Y\subset R$ (real axis) we proposed the following

DEFINITION 5.If

$$(\exists a>0)(\forall\delta>0)(\exists\varepsilon_0)(\forall\varepsilon<\varepsilon_0)$$

$$(\exists t_1, t_2\in T)((|t_1-t_2|<\delta)\wedge(|y_\varepsilon(t_1)-y_\varepsilon(t_2)|>a))$$

then the boundary layer phenomenon with width not less than a exists.

We have found two new kinds of this phenomenon (in the next section).

Consider the initial value problem

$$\varepsilon y_\varepsilon'(t)=f(t,y_\varepsilon(t)),\ t\in T=[0,b], \qquad (1)$$

$$y_\varepsilon(0)=y_0 \qquad (2)$$

and the corresponding degenerate equation

$$0=f(t,v(t)). \qquad (3)$$

DEFINITION 6.

If $(\exists k>0)(\forall t_1< t_2)\ (v(t_2) > v(t_1) + k(t_2 - t_1))$ then the function $v(t)$ is said to be linearly-increasing.

THEOREM 1 [1]. If 1) the equation (3) has a linearly-increasing (linearly-decreasing correspondingly) solution $v(t) \in C(T)$;

2) $sgn\, f(t,y)= -sgn\,(y- v(t))$

$(0 \leq t \leq b, y \in R)$ then $(\forall\delta>0)\ (\exists\varepsilon_0>0)\ (\forall\varepsilon<\varepsilon_0)$ $(y_\varepsilon(t) < v(t))$ (or : $y_\varepsilon(t) > v(t)$ correspondingly) $(\delta\leq t \leq b)$.

We called violation of these conditions "effect of non-monotonicity of solution of degenerate equation" which causes various phenomena in the theory of singularly perturbed differential equations.

We proposed the following

DEFINITION 7 [3]. If

$(\forall a>0)(\forall\delta>0)(\exists\varepsilon_0)\ (\forall\varepsilon<\varepsilon_0)(\ \exists t_1,$

$t_2\in T)((|\ t_1-t_2|<\delta)\wedge(|\ y_\varepsilon\ (t_1)-y_\varepsilon\ (t_2)|>a))$

thenthe phenomenon of deepening boundary layer exists.

For example, it takes placefor the initial value problem (1)-(2) with $f(t,y)=max\{1-ty,0\}$.

DEFINITION 8 [4]. If a dynamic system with εhas a periodic solution and its length tends to infinity as εtends to 0then such solution is said to be a *singular cycle.*

For example, the autonomous system

$\varepsilon x_\varepsilon'(t)=y_\varepsilon\ (t)-p(x_\varepsilon\ (t)),$

$p(x):=x(|x|-2)/(|x^2-1|+1),$

$y_\varepsilon'(t)=-x_\varepsilon\ (t)$

has the singular cycle.

References:

1. Kenenbaeva G. Theory and methodic of searching new effects and phenomena in the theory of perturbed differential and difference equations (in Russian). - Bishkek:"Ilim", 2012, 203 p.
2. Pankov P.S., Kenenbaeva G.M. The phenomenon of irgöö as the first example of dissipative system and its implementation on computer (in Kyrgyz) // Proceedings of the National Academy of Sciences of Kyrgyz Republic, no. 3, 2012, pp. 105-108.
3. Imanaliev M.I., Pankov P.S., Kenenbaeva G.M. Algorithm of qualitative investigation of singularly perturbed ordinary differential equations and autonomous systems of the second order, the phenomenon of singular cycle (in Russian) // Proceedings of the Russian Academy of Sciences, vol. 354, no. 6, 1997, pp. 733-735.
5. Imanaliev M.I., Pankov P.S., Kenenbaeva G.M. The phenomenon of the deepening boundary layer in the theory of singularly perturbed ordinary differential equations (in Russian)// Studies in integro-differential equations, issue 33. - Bishkek: "Ilim", 2004. - Pp. 15-19.

Рецензент: д.ф.-м.н., профессор Иманалиев Т.М.