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СПЕКТРИНИН ЧЕКИТИ ЭСЕЛҮҮ ЖАНА ТУРУКТУУ ЭМЕС ПАРАБОЛАЛЫК ТЕҢДЕМЕНИН ЧЫГАРЫЛЫШЫНЫН АСИМПТОТИКАСЫ

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АСИМПТОТИКА РЕШЕНИЯ ПАРАБОЛИЧЕСКОГО УРАВНЕНИЯ С КРАТКОЙ И НЕСТАБИЛЬНОЙ ТОЧКАМИ СПЕКТРА

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ASYMPTOTICS OF SOLVING A PARABOLIC EQUATION WITH BRIEF AND UNSTABLE POINT OF SPECTRUM

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Бул макалада сингулярдуу козголгон параболалык теңдеме спектринин чекити эселүү жана туруктуу эмес болгондогу маселе каралган. Асимптотиканы тургузуу А.С. Өмүралиев [1] тарабынан адаптацияланган сингулярдуу козголгон параболалык теңдемелерди регуляризациялоо методуна негизделген.

Негизги сөздөр: *сингулярдуу козголгон маселе, регуляризациялоо, регуляризациялоочу функциялар, өздүк маанилер, оператордун спектри, туруктуу эмес элемент.*

Рассматривается задача, когда сингулярно возмущенное параболическое уравнение имеет кратную и нестабильную точку спектра. Построение асимтотики основано на методе регуляризации для сингулярно возмущенных параболических задач, адаптированный А.С. Омуралиевым [1].

Ключевые слова: *сингулярно возмущенная задача, регуляризация, регуляризующие функции, собственные значения, спектр оператора, нестабильный элемент.*

In the article, the task of a singularly perturbed parabolic equation which has a multiple and unstable point of the spectrum is considered. The construction of the asymptotics based on the regularization method for singularly perturbed parabolic problems, adapted by A. Omuraliev [1].

Key words: *singularly perturbed problem, regularization, regularizing functions, eigenvalues, operator spectrum, unstable element.*

Consider the task

$$L_\varepsilon u \equiv \varepsilon\partial_t u - \varepsilon^2 a(x)\partial_x^2 u - L(t)u = f(x,y,t), \quad x,t \in \Omega, \tag{1}$$

$$u\Big|_{t=0} = h(x,y), \;\; u\Big|_{\partial\Omega} = 0, \tag{2}$$

Where $\Omega = \{(x,y,t):\;\; x,y \in (0,1),\; t \in (0,T)\}$, $\varepsilon > 0$ – the small parameter.

The task (1), (2) will be studied under the following assumptions:

1) $a(x) \in C^\infty[0,1]$, $f(x,t) \in C^\infty(\overline{\Omega})$,
2) $a(x) > 0$, $\forall x \in [0,1]$
3) $\{\lambda_i(t)\}$ the spectrum of a self-ad joint operator of a simple structure *L(t),* for each $t \in [0,T]$ supplies conditions

$$\lambda_1(t) \equiv \lambda_2(t) \equiv \cdots \equiv \lambda_p(t) < \lambda_{p+1}(t) < \lambda_{p+2}(t) < \cdots < \lambda_n(t) < \cdots < 0$$

where $\lambda_i(t) \neq 0,\ \forall\, i = \overline{p+2, n},$ and the unstable element of the spectrum of the operator is representable in the form

$$\lambda_{p+1}(t) = tq(t), \qquad q(t) < 0, \qquad \forall\ t \in [0, T],$$

4. The condition for matching the initial and boundary conditions is in progress:
h(0,y) = h(1,y) = 0.

We generalize the results of [1] to the case when the limit operator has a multiple and unstable spectrum.

П.1. *The regularization of the task (1),(2).* We extend the operator L_ε, in order to inculcate regularizing functions:

$$\tau_j = \frac{1}{\varepsilon}\int_0^t \lambda_j(s)ds \equiv \frac{\varphi_j(t)}{\varepsilon}, \quad \tau_{p+1} = \frac{1}{\sqrt{\varepsilon}}\left(-2\int_0^t \lambda_{p+1}(s)ds\right)^{1/2} \equiv \frac{1}{\sqrt{\varepsilon}}\varphi_{p+1}(t), \tag{3}$$

$$j = p, p+2, p+3, \qquad \eta = \frac{t}{\varepsilon^2}, \qquad \xi_l = \frac{(-1)^{l-1}}{\sqrt{\varepsilon}}\int_{l-1}^{x}\frac{ds}{\sqrt{a(s)}} \equiv \frac{\psi_l(x)}{\sqrt{\varepsilon}}, \ \ \zeta_l = \frac{\psi_l(x)}{\sqrt{\varepsilon^3}},$$

and instead of the desired function u (x, y.t, ε) we consider the extended function

$\tilde{u}(M,\varepsilon), M = (x, y, t, \xi, \tau, \eta),\ \xi = (\xi_1, \xi_2),\ \tau = (\tau_1, \tau_2)$, such, that

$$\tilde{u}(M,\varepsilon)\,\big|_{\theta=\alpha(x,t,\varepsilon)} \equiv u(x, y, t, \varepsilon), \tag{4}$$

$$\theta = (\xi, \tau, \eta), \qquad \alpha(x,t,\varepsilon) = \left(\frac{\varphi(t)}{\varepsilon}, \frac{\varphi_{p+1}(t)}{\sqrt{\varepsilon}}, \frac{\psi(x)}{\sqrt{\varepsilon}}, \frac{\psi(x)}{\sqrt{\varepsilon^3}}\right),$$

$$\varphi(t) = \left(\varphi_1, \varphi_2, \dots, \varphi_p, \varphi_{p+2}, \dots\right), \qquad \psi(x) = \left(\psi_1(x), \psi_2(x)\right).$$

Find out

$$\partial_t u \equiv \left(\partial_t\tilde{u} + \frac{1}{\varepsilon^2}\partial_\eta\tilde{u} + \frac{1}{\varepsilon}\sum_{j=p(j\neq p+1)}^{\infty}\lambda_j(t)\partial_{\tau_j}\tilde{u} + \frac{1}{\sqrt{\varepsilon}}\varphi'_{p+1}(t)\partial_{\tau_{p+1}}\tilde{u}\right)_{\theta=\alpha(x,t,\varepsilon)},$$

$$\partial_x u \equiv \left(\partial_x\tilde{u} + \frac{1}{\sqrt{\varepsilon}}\sum_{l=1}^{2}\psi_l(x)\partial_{\xi_l}\tilde{u} + \frac{1}{\sqrt{\varepsilon^3}}\sum_{l=1}^{2}\psi'_l(x)\partial_{\zeta_l}\tilde{u}\right)_{\theta=\alpha(x,t,\varepsilon)}, \tag{5}$$

$$\partial_x^2 u \equiv (\partial_x\tilde{u} + \sum_{l=1}^{2}\left[\left(\frac{\psi'_l(x)}{\sqrt{\varepsilon}}\right)^2\partial^2_{\xi_l}\tilde{u} + 2\frac{\psi'_l(x)}{\sqrt{\varepsilon}}\partial^2_{x\xi_l}\tilde{u} + \frac{\psi''_l(x)}{\sqrt{\varepsilon}}\partial_{\xi_l}\tilde{u}\right] +$$

$$+\sum_{l=1}^{2}\left[\left(\frac{\psi'_l(x)}{\sqrt{\varepsilon^3}}\right)^2\partial^{l_2}_{\zeta_l}\tilde{u} + 2\frac{\psi'_l(x)}{\sqrt{\varepsilon^3}}\partial^2_{x\xi_l}\tilde{u} + \frac{\psi''_l(x)}{\sqrt{\varepsilon^3}}\partial_{\xi_l}\tilde{u}\right])\ _{\theta=\alpha(x,t,\varepsilon)}.$$

Using the representation of the operator L(t)through the projectors:

$$L(t) = \sum_{j=1}^{\infty} \lambda_j(t) P_j(t),$$

whereP_j proper projectors corresponding to $\lambda_j(t)$, on the basis of (4), (5), from (1), (2) and we obtain the extended task

$$\tilde{L}_\varepsilon \tilde{u} \equiv \varepsilon \partial_t \tilde{u} + \frac{1}{\varepsilon} \partial_\eta \tilde{u} + D_\lambda \tilde{u} + \sqrt{\varepsilon} \varphi'_{p+1}(t) \partial_{\tau_{p+1}} \tilde{u} - \varepsilon^2 a(x) \partial^2{}_x \tilde{u} - \frac{1}{\varepsilon} \Delta_\zeta \tilde{u} - \sqrt{\varepsilon} D_\zeta \tilde{u}$$

$$-\varepsilon \Delta_\xi \tilde{u} - \sqrt{\varepsilon^3} D_\xi \tilde{u} - \sum_{j=p}^{\infty} \lambda_j^*(t) P_j(t) \tilde{u}(M, \varepsilon) = f(x, y, t), \qquad M \in Q$$

$$\tilde{u} \,\Big|_{t=0, \tau=\eta=0} = h(x, y), \qquad \tilde{u} \,\Big|_{\partial Q} = 0,$$

or, regrouping the terms, we rewrite

$$\tilde{L}_\varepsilon \tilde{u}(M, \varepsilon) \equiv \varepsilon \partial_t \tilde{u} + \frac{1}{\varepsilon} \partial_\eta \tilde{u} + D_\lambda \tilde{u} - \varepsilon \Delta_\xi u - \sum_{j=1}^{\infty} \lambda_j(t) P_j(t) \tilde{u}(M, \varepsilon) - \frac{1}{\varepsilon} \Delta_\zeta \tilde{u} -$$

$$-\sqrt{\varepsilon} D_\zeta \tilde{u} + \sqrt{\varepsilon} \varphi'_{p+1}(t) \left[\partial_{\tau_{p+1}} \tilde{u}\right] - \sqrt{\varepsilon^3} D_\xi \tilde{u} - \varepsilon^2 D_x \tilde{u}(M, \varepsilon) = f(x, y, t), M \in Q \quad (6)$$

$$\tilde{u} \,\Big|_{t=0, \tau=\eta=0} = h(x), \qquad \tilde{u}(M, \varepsilon) \,\Big|_{\partial Q} = 0,$$

$$D_\lambda \equiv \sum_{j=p(j \neq p+1)}^{\infty} \lambda_j(t) \partial_{\tau_j}, \quad \Delta_\xi \equiv \partial_{j_1}^2 + \partial_{j_2}^2, \quad L_x \equiv a(x) \partial_x^2,$$

$$D_{x, \xi_l} = a(x) \left[2\psi_l'(x) \partial_{x, \xi_l} + \psi_l''(x)\right].$$

By the solution of the extended task (6) we will define in the form

$$\tilde{u}(M, \varepsilon) = \sum_{i=-1}^{\infty} \varepsilon^{\frac{i}{2}} \sum_{k=1}^{\infty} b_k(y, t) \Bigg(v_{i,k}(x, t) + \sum_{l=1}^{2} u_{i,k}^l(N_l) +$$

$$+ \sum_{j=p}^{\infty} \left[C_{i,k}^j(x, t) + \sum_{l=1}^{2} Z_{i,k}^{l,j}(x, t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right] \exp(\tau_j) +$$

$$\left[q_{i,k}(x, t) + \sum_{l=1}^{2} \omega_{i,k}^l(x, t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right] \gamma(\tau_{p+1}) \Bigg) \equiv V_1 + V_2 + V_3, \qquad (7)$$

where $\gamma(\tau_{p+1})$ is the solution of the task.

$$\gamma'(\tau_{p+1}) + \tau_{p+1} \gamma(\tau_{p+1}) = 1, \ \gamma(0) = 0 \qquad (*)$$

Taking into account (*), and $\sqrt{\varepsilon} \varphi'_{p+1}(t) \tau_{p+1} = -\lambda_{p+1}(t)$ we have

$$\sqrt{\varepsilon}\varphi'_{p+1}(t)\partial_{\tau_{p+1}}\gamma(\tau_{p+1}) = \sqrt{\varepsilon}\varphi'_{p+1}(t)[1-\tau_{p+1}\gamma(\tau_{p+1})] =$$

$$= \varepsilon\varphi'_{p+1}(t) + \lambda_{p+1}(t)\gamma(\tau_{p+1}).$$

In view of the latter, we calculate the action of the operator $\tilde{L}_\varepsilon$ on the function

$$V_3 = \sum_{i=-1}^{\infty} \varepsilon^{i/2} \sum_{k=1}^{\infty} b_k(y,t)\left[q_{i,k}(x,t) + \sum_{l=1}^{2} \omega_{i,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{\tau}}\right)\right]\gamma(\tau_{p+1}),$$

and we got

$$\tilde{L}_\varepsilon V_3 = \sum_{i=-1}^{\infty} \varepsilon^{\frac{i}{2}} \sum_{k=1}^{\infty}\left[-P_\lambda + \sqrt{\varepsilon}\varphi'_{p+1}(t)\partial_{\tau_{p+1}} + \varepsilon(\partial_t - \Delta_\xi) - \varepsilon^2 L_x\right]\times$$

$$\times\left[q_{i,k}(x,t) + \sum_{e=1}^{2} \omega_{i,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma(\tau_{p+1}) b_k(y,t) =$$

$$= \sum_{i=-1}^{\infty} \varepsilon^{\frac{i}{2}} \sum_{k=1}^{\infty}\left\{\left[\left(-\lambda_k(t) + \lambda_{p+1}(t)\right)\gamma(\tau_{p+1}) + \sqrt{\varepsilon}\varphi'_{p+1}(t)\right]\times\right.$$

$$\times\left[q_{i,k}(x,t) + \sum_{l=1}^{2} \omega_{i,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right] b_k(y,t) +$$

$$+\varepsilon\left[\left(q_{i,k}(x,t)b_k(y,t)\right)'_t + \sum_{l=1}^{2}\left(\omega_{i,k}^{l}(x,t)b_k(y,t)\right)'_t erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma(\tau_{p+1}) -$$

$$-\varepsilon^{3/2}\sum_{l=1}^{2} D_{x,\xi_l}\left(\omega_{i,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)\gamma(\tau_{p+1}) b_k(y,t) -$$

$$\left.-\varepsilon^2 L_x\left[q_{i,k}(x,t) + \sum_{l=1}^{2} \omega_{i,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma(\tau_{p+1}) b_k(y,t)\right\}. \qquad (8)$$

We make the substitution $\theta_j = \tau_j$, $\theta_{p+1} = -\frac{\tau_{p+1}^2}{2}$ and calculate the action of the operator

$$\left(D_\lambda + \sqrt{\varepsilon}\varphi'_{p+1}(t)\partial_{\tau_{p+1}}\right)u_i = \sum_{j=1(j\neq p+1)}^{\infty} \lambda_j(t)\partial_{\theta_j}u_i + \lambda_{p+1}(t)\partial_{\theta_{p+1}}u_i =$$

$$= \sum_{j=1}^{\infty}\lambda_j(t)\partial_{\theta_j}u_i, \qquad (9)$$

the ratio is taken into account here

$$\sqrt{\varepsilon}\varphi'_{p+1}(t)\partial_{\tau_{p+1}}u_i = -\sqrt{\varepsilon}\varphi'_{p+1}(t)\tau_{p+1}\partial_{\theta_{p+1}}u_i = -\sqrt{\varepsilon}\varphi'_{p+1}(t)\frac{\varphi_{p+1}(t)}{\sqrt{\varepsilon}}\partial_{\theta_{p+1}}u_i = \varphi_{p+1}(t)\varphi'_{p+1}(t)\partial_{\theta_{p+1}}u_i$$
$$= \lambda_{p+1}(t)\partial_{\theta_{p+1}}u_i.$$

Next, we calculate the action of the operator $\tilde{L}_\varepsilon$ on the function V$_2$, V$_1$:

$$\tilde{L}_\varepsilon V_2 = \sum_{i=-1}^{\infty} \varepsilon^{\frac{i}{2}} \sum_{k=1}^{\infty} \left\{ \sum_{j=p}^{\infty} \left[C_{i,k}^{j}(x,t) + \sum_{l=1}^{2} Z_{i,k}^{l,j}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right] \times \right.$$

$$\times \left[\lambda_j(t) b_k(y,t) e^{\theta_j} - \lambda_k(t) b_k(y,t) e^{\theta_j} \right] +$$

$$+\varepsilon \sum_{j=p}^{\infty} \left[\partial_t C_{i,k}^{j}(x,t) b_k(y,t) + \sum_{\nu=1}^{\infty} \alpha_{k,\nu}(t) b_\nu(y,t) C_{i,k}^{j}(x,t) + \right.$$

$$\left. + \sum_{l=1}^{2} \left(\partial_t Z_{i,k}^{l,j}(x,t) b_k(y,t) + \sum_{\nu=1}^{\infty} \alpha_{k,\nu}(t) b_\nu(y,t) Z_{i,k}^{l,j}(x,t) \right) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right] \exp(\theta_j) -$$

$$-\sqrt{\varepsilon^3} \sum_{j=p}^{\infty} \sum_{l=1}^{2} D_{\xi,l} \left(Z_{i,k}^{l,j}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right) \exp(\theta_j)\, b_k(y,t) -$$

$$\left. -\varepsilon^2 \sum_{j=p}^{\infty} \left(L_x C_{i,k}^{j}(x,t) + \sum_{l=1}^{2} L_x \left(Z_{i,k}^{l,j}(x,t) \right) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) \right) \exp(\theta_j)\, b_k(y,t) \right\}, \quad (10)$$

$$\tilde{L}_\varepsilon V_1 = \sum_{i=1}^{\infty} \varepsilon^{\frac{i}{2}} \sum_{k=1}^{\infty} \left\{ \frac{1}{\varepsilon} T_1 u_{i,k}^{l}(N_l) b_k(y,t) - \right.$$

$$-\sum_{l=1}^{2} \lambda_k(t) u_{i,k}^{l}(N_l) b_k(y,t) - \sqrt{\varepsilon} \sum_{l=1}^{2} D_\zeta u_{i,k}^{l}(N_l) +$$

$$+\varepsilon \sum_{l=1}^{2} \left[\partial_t \left(u_{i,k}^{l} b_k \right) - \varepsilon L_x u_{i,k}^{l} b_k(y,t) \right] - \lambda_k(t) V_{i,k}(x,t) b_k(y,t) -$$

$$\left. -\varepsilon \partial_t \left(V_{i,k}(x,t) b_k(y,t) \right) - \varepsilon^2 L_x V_{i,k}(x,t) b_k(y,t) \right\},$$

$$T_1 \equiv \partial_\eta - \Delta_\zeta \,. \quad (11)$$

Taking into account the above calculations (8) - (11) the extended equation (6) is written

$$\tilde{L}_\varepsilon \tilde{u} \equiv \sum_{i=-1}^{\infty} \varepsilon^{i\backslash 2} \sum_{k=1}^{\infty} \left\{ \sum_{l=1}^{2} \left[\frac{1}{\varepsilon} T_1 u_{i,k}^{l}(N_l) - \lambda_k(t) u_{i,k}^{l}(N_l)\, b_k(y,t) - \sqrt{\varepsilon} D_\zeta u_{i,k}^{l}(N_l) + \right. \right.$$

$$+\varepsilon \sum_{\mu=1}^{\infty} \alpha_{k,\mu}(t) b_\mu(y,t)\, u_{i,k}^{l}(N_l) - \lambda_k(t) V_{i,k}(x,t) b_k(y,t) +$$

$$+\varepsilon \left(\partial_t V_{i,k}(x,t) b_k(y,t) + \sum_{\mu=1}^{\infty} \alpha_{k,\mu}(t) b_\mu(y,t) V_{i,k}(x,t) \right) + \varepsilon^2 L_x (V_{i,k}(x,t)) b_k(y,t) +$$

$$+\sum_{j=p}^{\infty}[\,(\lambda_j(t)-\lambda_k^*(t)\left(C_{i,k}^{j}(x,t)+\sum_{l=1}^{2}Z_{i,k}^{l,j}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)b_k(y,t)+$$

$$+\varepsilon\left(\partial_t C_{i,k}^{j}(x,t)b_k(y,t)+\sum_{\mu=1}^{\infty}\alpha_{k,\mu}(t)b_\mu(y,t)C_{i,k}^{j}(x,t)\right)]+$$

$$+\sum_{k=1}^{\infty}[\left(\lambda_{p+1}-\lambda_k(t)\right)q_{i,k}(x,t)\gamma\left(\tau_{p+1}\right)b_k(y,t)+\sqrt{\varepsilon}\varphi_{p+1}'(t)q_{i,k}(x,t)+$$

$$+\varepsilon\left(\partial_t q_{i,k}(x,t)+\sum_{\mu=1}^{\infty}\alpha_{\mu,k}(t)q_{i,\mu}(x,t)\right)b_k(y,t)\gamma\left(\tau_{p+1}\right)-$$

$$-\varepsilon^2 L_x\left(q_{i,k}(x,t)\right)b_k(y,t)\gamma\left(\tau_{p+1}\right)]+$$

$$+\varepsilon\sum_{l=1}^{2}\left(\partial_t Z_{i,k}^{l,j}(x,t)+\sum_{\mu=1}^{\infty}\alpha_{\mu,k}(t)Z_{i,\mu}^{l,j}(x,t)\right)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)b_k(y,t)-$$

$$-\sqrt{\varepsilon^3}\sum_{l=1}^{2}D_{x,\xi_l}\left(Z_{i,k}^{l,j}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)b_k(y,t)-$$

$$-\varepsilon^2\left(L_x C_{i,k}^{j}(x,t)+\sum_{l=1}^{2}L_x\left(Z_{i,k}^{l,j}(x,t)\right)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)b_k(y,t)\right)\exp(\theta_j)+$$

$$+\sum_{k=1}^{\infty}[\left(\lambda_{p+1}(t)-\lambda_k(t)\right)q_{i,k}(x,t)\gamma\left(\tau_{p+1}\right)b_k(y,t)+\sqrt{\varepsilon}\varphi_{p+1}'(t)q_{i,k}(x,t)+$$

$$+(\partial_t q_{i,k}(x,t)+\sum_{\mu=1}^{\infty}\alpha_{\mu,k}(t)q_{i,\mu}(x,t))b_k(y,t)\gamma\_(\tau_{p+1})-\varepsilon^2 L_x(q_{i,k}(x,t))+$$

$$+\varepsilon\left(b_k(y,t)\gamma\left(\tau_{p+1}\right)\right]+$$

$$+\sum_{k=1}^{\infty}\sum_{l=1}^{2}\left[\left(\lambda_{p+1}(t)-\lambda_k(t)\right)\omega_{i,k}^{l}(x,t)\gamma\left(\tau_{p+1}\right)b_k(y,t)+\sqrt{\varepsilon}\varphi_{p+1}'(t)\omega_{i,k}^{l}(x,t)\right]\times$$

$$\times erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)b_k(y,t)+$$

$$+\varepsilon\sum_{k=1}^{\infty}\sum_{l=1}^{2}\left[\partial_t\omega_{i,k}^{l}(x,t)+\sum_{\mu=1}^{\infty}\alpha_{\mu,k}(t)\omega_{i,\mu}^{l}(x,t)\right]b_k(y,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\gamma\left(\tau_{p+1}\right)-$$

$$-\sqrt{\varepsilon^3}\sum_{k=1}^{\infty}\sum_{l=1}^{2}\left[D_{x,\xi_l}\left(\omega_{i,k}^{l}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)b_k(y,t)\gamma(\tau_{p+1})\right]-$$

$$-\varepsilon^2\sum_{k=1}^{\infty}\sum_{l=1}^{2}L_x\left(\omega_{i,k}^{l}(x,t)\right)b_k(y,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\gamma(\tau_{p+1})=\sum_{k=1}^{\infty}f_k(x,t)b_k(y,t).$$

$$f(x,y,t)=\sum_{k=1}^{\infty}f_k(x,t)b_k(y,t),\quad \alpha_{k,\mu}(t)=(\partial_t b_k,b_\mu).$$

The coefficients for the same powers of ε is equated here:

$$\sum_{l=1}^{2}T_{1,l}u_{-1,k}^{l}(N_l)=0,\qquad l=1,2;\qquad \sum_{l=1}^{2}T_1u_{0,k}^{l}(N_l)=0,$$

$$\sum_{l=1}^{2}T_{1,2}u_{1,k}^{l}(N_l)=$$

$$=-\sum_{j=p}^{\infty}\left(\lambda_j(t)-\lambda_k(t)\right)\left[C_{-1,k}^{j}(x,t)+\sum_{l=1}^{2}Z_{-1,k}^{l,j}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]exp(\theta_j)-$$

$$-\left(\lambda_{p+1}(t)-\lambda_k^{*}(t)\right)\left[q_{-1,k}(x,t)+\sum_{l=1}^{2}\omega_{-1,k}^{l}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma(\tau_{p+1})+$$

$$+\lambda_k(t)\left(\sum_{l=1}^{2}u_{-1,k}^{l}(N_l)+V_{-1,k}(x,t)\right);$$

$$T_{1,l}u_{2,k}^{l}(N_l)=\lambda_k(t)\left(\sum_{l=1}^{2}u_{0,k}^{l}(N_l)+V_{0,k}(x,t)\right)-$$

$$-\sum_{j=p}^{\infty}(\lambda_j(t)-\lambda_k(t))\left[C_{0,k}^{j}(x,t)+\sum_{l=1}^{2}Z_{0,k}^{l,j}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\exp(\theta_j)-$$

$$-(\lambda_{p+1}(t)-\lambda_k(t))\left[q_{0,k}(x,t)+\sum_{l=1}^{2}\omega_{0,k}^{l}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma(\tau_{p+1})-$$

$$-\varphi_{p+1}'(t)q_{-1,k}(x,t)-$$

$$-\varphi_{p+1}'(t)\sum_{l=1}^{2}\omega_{-1,k}^{l,p+1}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)+f_k(x,t)+\sum_{l=1}^{2}D_\zeta u_{-1,k}^{l},$$

$$\sum_{l=1}^{2} T_{1,l} u_{3,k}^{l}(N_l) = \lambda_k(t)\left(u_{1,k}^{l}(N_l) - V_{1,k}(x,t)\right) -$$

$$-\sum_{j=p}^{\infty}\left(\lambda_j(t) - \lambda_k(t)\right)\left[C_{1,k}^{j}(x,t) + \sum_{l=1}^{2} Z_{1,k}^{l,j}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\exp(\theta_j)$$

$$-\left(\lambda_{p+1}(t) - \lambda_k(t)\right)\left[q_{i,k}(x,t) + \sum_{l=1}^{2} \omega_{1,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right]\gamma\left(\tau_{p+1}\right) -$$

$$-\varphi'_{p+1}(t) q_{0,k}(x,t) + \sum_{l=1}^{2} D_{x_\lambda \zeta_l} u_{0,k}^{l}(N_l) - \sum_{j=p}^{\infty}\{\partial_t C_{-1,k}^{j}(x,t) -$$

$$-\left[\sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) C_{-1,k}^{j}(x,t) +\right.$$

$$\left.+\sum_{l=1}^{2}\left(\partial_t Z_{-1,k}^{l,j}(x,t) + \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) Z_{-1,\mu}^{l,j}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)\right]\}\exp\left(\theta_j\right) -$$

$$-\left[\partial_t q_{-1,k}(x,t) + \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) q_{-1,\mu}(x,t)\right]\gamma\left(\tau_{p+1}\right) -$$

$$-\sum_{l=1}^{2}\left[\partial_t \omega_{-1,k}^{l}(x,t) + \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) \omega_{-1,\mu}^{l}(x,t)\right] erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\gamma\left(\tau_{p+1}\right) -$$

$$\sum_{l=1}^{2}(\varphi'_{p+1}(t) \omega_{0,k}^{l}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) + \left[\partial_t u_{-1,k}^{l} + \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) u_{-1,\mu}^{l}\right]) +$$

$$+\partial_t V_{-1,k}(x,t) + \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) V_{-1,\mu}(x,t)$$

From the obtained correlations, turning them into identities, for arbitrary

$C_{-1,k}^{k}(x,t)$, $Z_{-1,k}^{l,k}(x,t)$, $q_{-1,p+1}(x,t)$, $\omega_{-1,p+1}^{l}(x,t)$, $V_{-1,p+1}^{0}(x,t)$, we get:

$$T_{1,l} u_{-1,k}^{l}(N_l) \equiv \partial_\eta u_{-1,k}^{l}(N_l) - \partial_{\zeta_l}^{2} u_{-1,k}^{l}(N_l) = 0, \qquad T_{1,l} u_{0,k}^{l}(N_l) = 0,$$

$$T_{1,l} u_{1,k}^{l}(N_l) = \lambda_k(t) u_{-1,k}^{l}(N_l), \quad C_{-1,k}^{j}(x,t) = 0, \quad Z_{-1,k}^{l,j}(x,t) = 0, \forall\, j \neq k,$$

$$\omega_{-1,k}^{l,p+1}(x,t) = 0, V_{-1,k}(x,t) = 0, \qquad \forall k \neq p+1,$$

$$\nu_{-1,p+1}(x,t) = \begin{cases} 0, & \forall\, t \in [0,\tau] \\ \nu_{-1,p+1}^{0}(x), & \forall\, t = 0, \end{cases}$$

Taking into account these ratios, in the next step we get:

$$T_{1,l}u_{2,k}^{l}(N_l) = \lambda_k(t)u_{0,k}^{l}(N_l) - \varphi'_{p+1}(t)\omega_{-1,k}^{l}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right), \quad (15)$$

$$\lambda_k(t)V_{0,k}(x,t) - \varphi'_{p+1}(t)q_{-1,k}(x,t) + f_k(x,t) = 0, C_{0,k}^{j}(x,t) = 0, Z_{0,k}^{l,j}(x,t) = 0,$$

$$\forall j \neq k, \qquad q_{0,k}(x,t) = 0, \omega_{0,k}^{l}(x,t) = 0, \forall\, k \neq p+1, D_{x,\zeta_l}u_{-1,k}^{l}(N_l) = 0,$$

The functions $C_{0,k}^{k}(x,t),\ Z_{0,k}^{l,k}(x,t),\ q_{0,p+1}(x,t),\ \omega_{0,p+1}^{l}(x,t)$ − are arbitrary.

The function $erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)$, enters the right side of the first equation, and the functions $u_{2,k}^{l}(N_l)$depends on the variables $N_l = (x,t,\eta,\zeta_l)$, therefore we pass to $erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)$ to such variables, so as

$$\left(\frac{\xi_l}{2\sqrt{t}}\right) = \frac{\varphi_l(t)}{2\sqrt{\varepsilon}\sqrt{t}} = \frac{\varphi_l(t)}{2\sqrt{\varepsilon^3}\sqrt{\frac{t}{\varepsilon^2}}} = \frac{\frac{\varphi_l(t)}{\sqrt{\varepsilon^3}}}{2\sqrt{\frac{t}{\varepsilon^2}}} = \frac{\zeta_l}{2\sqrt{\eta}}, \text{то } erfc\left(\frac{\xi_l}{2\sqrt{t}}\right) = erfc\left(\frac{\zeta_l}{2\sqrt{\eta}}\right).$$

After such a replacement, the solution of this equation under boundary conditions

$u_{2,k}^{l}(N_l)\,\big|_{t=0,\eta=0} = 0,\ u_{2,k}^{l}(N_l)\,\big|_{x=l-1,\xi_l=0} = -V_{2,k}(l-1,t)$, can be written in the form

$$u_{2,k}^{l}(N_l) = d_{2,k}^{l}(x,t)erfc\left(\frac{\zeta_l}{2\sqrt{\eta}}\right) + I(N_l).$$

Here $I(N_l)$ is a well-known integral having the following estimatation:

$$|I(N_l)| < c\exp\left(-\frac{\zeta_l^2}{8\eta}\right).$$

Consider the second equation from (15), so $q_{-1,k}(x,t) = 0,\ \forall\, k \neq p+1$, we have

$$-\lambda_{p+1}(t)V_{0,p+1}(x,t) - \varphi'_{p+1}(t)q_{-1,p+1}(x,t) + \varphi_{p+1}(x,t) = 0 \quad (16)$$

Or $\lambda_k(t)V_{0,k}(x,t) + f_k(x,t) = 0,\ \forall\, k \neq p+1, k \geq 1, \lambda_k(t) \neq 0, \forall\, k \neq p+1$, then

$$V_{0,k}(x,t) = -\frac{f_k(x,t)}{\lambda_k(t)}, \forall\, k \neq p+1, \quad (17)$$

For *k=p+1*, the eigenvalue of $\lambda_{p+1}(t) = 0, \forall\, t = 0$, therefore for t = 0 the function $q_{-1,p+1}(x,t)$, (x, t) is defined in the form:

$$q_{-1,p+1}(x,0) = \frac{f_{p+1}(x,0)}{\varphi'_{p+1}(0)}, \quad (18)$$

in this case the function

$$V_{0,p+1}(x,t) = \begin{cases} \dfrac{\varphi'_{p+1}(t)q_{-1,p+1}(x,t) + f_{p+1}(x,t)}{\lambda_{p+1}(t)} & under\ t \in (0,\tau] \\ V_{p+1}^{0}(x)\ \ an\ arbitrary\ function\ with\ \ t = 0 \end{cases} \quad (19)$$

Thus, it is established that equation (16) is solvable.

To satisfy the relation $D_{x,\zeta_l} u^l_{-1,k}(N_l) = 0$, the solution of equation (14) concerning to $u^l_{-1,k}(N_l)$, supplying the condition:

$$u^l_{-1,k}\big|_{t=0,\tau=0} = 0, \qquad u^l_{-1,k}(N_l)\big|_{x=l-1} = -V_{-1,k}(l-1,t),$$

is written in the form

$$u^l_{-1,k}(N_l) = d^l_{-1,k}(x,t) erfc\left(\frac{\xi_l}{2\sqrt{\eta}}\right), \qquad d^l_{-1,k}(l-1,t) = V_{-1,k}(l-1,t),$$

where an arbitrary function $d_{-1,k}(x,t)$ is chosen so that

$$D_{x,\zeta} u^l_{-1,k}(N_l) \equiv \left[2\psi_l'(x)\partial_x d_{-1,k}(x,t) + \psi_l''(x) d_{-1,k}(x,t)\right]\partial_\zeta\left(erfc\left(\frac{\zeta_l}{2\sqrt{\eta}}\right)\right) = 0,$$

or

$$2\psi_l'(x)\partial_x d_{-1,k}(x,t) - \psi_l''(x) d_{-1,k}(x,t) = 0. \qquad (20)$$

Solving this equation under the initial condition

$d_{-1,k}(l-1,t) = -V_{-1,k}(l-1,t) = 0$ we will ensure the validity of the correlations (15).

Consider equation (13) in i = 3, we set:

$$-\lambda_k^*(t) V_{1,k}(x,t) - \varphi'_{p+1}(t) q_{0,k}(x,t) - \partial_t V_{-1,k}(x,t) - \sum_{\mu=1}^{\infty} \alpha_{\mu,k}(t) V_{-1,\mu}(x,t) = 0,$$

$$\partial_t C^j_{-1,j} + \alpha_{j,j}(t) C^j_{-1,j}(x,t) = 0, \qquad \partial_t Z^{l,j}_{-1,j} + \alpha_{j,j}(t) Z^{l,j}_{-1,j}(x,t) = 0,$$

$$\left(\lambda_j(t) - \lambda_k^*(t)\right) C^j_{1,k}(x,t) = \partial_t C^j_{-1,k} + \sum_{\mu=1(\mu\neq k)}^{\infty} \alpha_{\mu,k}(t) C^j_{-1,\mu}(x,t), \ \ j \neq k,$$

$$(\lambda_k^*(t) - \lambda_j(t)) Z^{l,j}_{1,k}(x,t) = \partial_t Z^j_{-1,k} + \sum_{\mu=1(\mu\neq k)}^{\infty} \alpha_{\mu,k}(t) Z, \ \ j \neq k, \qquad (21)$$

$$(\lambda_{p+1}(t) - \lambda_k^*(t)) q_{1,k}(x,t) = \sum_{\mu=1(\mu\neq p+1)}^{\infty} \alpha_{\mu,k}(t) q_{-1,\mu}(x,t) + \partial_t q_{-1,k}(x,t), k \neq p+1,$$

$$\partial_t q_{-1,p+1} + \alpha_{p+1,p+1}(x,t) = 0,$$

$$\left(\lambda_{p+1} - \lambda_k^*\right)\omega^l_{1,k}(x,t) = \partial_t \omega^l_{-1,k} + \sum_{\mu=1(\mu\neq p+1)}^{\infty} \alpha_{\mu,k}(t) \omega^l_{-1,k}(x,t), \ \ \forall\, k \neq p+1,$$

$$\partial_t \omega^l_{-1,p+1} + \alpha_{p+1,p+1}(t) \omega^l_{-1,p+1}(x,t) = 0.$$

On the base (14), (15) $V_{-1,k}(x,t)=0,\ q_{0,k}(x,t)=0, \forall\, k\neq p+1$, therefore the first equitation in (21) is written:

$$\lambda_{p+1}(t)V_{1,p+1}(x,t)-\varphi'_{p+1}(t)q_{0,p+1}(x,t)-\partial_t V_{-1,p+1}(x,t)-$$

$$-\alpha_{p+1,p+1}(t)V_{-1,p+1}(x,t)=0,$$

$$-\lambda_k^*(t)V_{1,k}(x,t)=0, \qquad \forall\, k\neq p+1, k=1,2\,...$$

Hence, for *t=0* и *k=p+1* we get:

$$q_{0,p+1}(x,t)=-\frac{\beta_{p+1}(x,0)}{\varphi'_{p+1}(0)}, \qquad \beta_{p+1}(x,t)=\alpha_{p+1,p+1}(t)V^0_{-1,p+1}(x),$$

and for $t\neq 0$

$$V_{1,p+1}(x,t)=\begin{cases}-\dfrac{1}{\lambda_{p+1}(t)}\left[\varphi'_{p+1}(t)q_{0,p+1}(x,t)+\alpha_{p+1,p+1}(t)V^0_{-1,p+1}(x)\right]\\ \quad V^0_{1,p+1}(x),\ \forall\, t=0, \qquad arbitrary\ function.\end{cases}$$

In $k\neq p+1,\ V_{1,k}(x,t)=0$.

In order to solve differential equitation concerning $C^j_{-1,j}(x,t),\ Z^{l,j}_{-1,j}(x,t),\ \omega^{l,p+1}_{-1,p+1}(x,t)$, from (21) the initial conditions are given. From the boundary conditions (6), on the basis of (7) we obtain:

$$u^l_{i,k}(N_l)\,\big|_{t=\eta=0}=0, C^j_{i,k}(x,t)\,\big|_{t=0}=\begin{cases}h_k(x),\ \forall\, i=0,\\ -V_{i,k}(x,0),\ i\neq 0, Z^{l,j}_{i,k}(x,t)\,\big|_{t=0}=\tilde{Z}^{l,j}_{i,k}(x),\end{cases}$$

$$u^l_{i,k}(N_l)\,\big|_{x=l-1,\zeta_l=0}=-V_{i,k}(l-1,t), \qquad Z^{l,j}_{i,k}(x,t)\,\big|_{x=l-1}=C^j_{i,k}(l-1,t),$$

$$\omega^l_{i,k}(x,t)\,\big|_{x=l-1}=-q_{i,k}(l-1,t),$$

here $\tilde{Z}^{l,j}_{l,k}(x)$ – arbitrary functions, they are defined from the equitation

$$D_{x,\xi_l}\left(Z^{l,j}_{l,k}(x,t)erfc\left(\frac{\xi_l}{2\sqrt{t}}\right)\right)=0,$$

and the arbitrary function $d^l_{i,k}(x,t)\ including\ into\ u^l_{i,k}(N_l)$ is defined on the base of equitation of similarly equation of (20). Equitation

$$\partial_t q_{-1,p+1}+\alpha_{p+1,p+1}(t)q_{-1,p+1}(x,t)=0,$$

we solve under the initial condition (18).

Further, repeating the above-described process, we define all the functions included in the partial sum of the series of (7) and establish the asymptotic character of the constructed solution, in other words, the following is proved.

Theorem: *The conditions 1) -3) are supposed to be done. Then the restriction of* $\theta = \chi(x, t, \varepsilon)$ *the partial sum of the series (18), constructed in the way described above is an asymptotic solution of the task (1), (2), i.e. the estimation is fair*

$$|u(x, y, t, \varepsilon) - u_{\varepsilon n}(x, y, t, \chi(x, t, \varepsilon))| < C\varepsilon^{\frac{n+1}{2}}, \qquad \forall\, n = 0,1,2, \dots$$

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