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СЫЗЫКТУУ LC-ЧЫНЖЫРЛАРЫНДАГЫ РЕЗОНАНТТЫК ЖАНА АНТИРЕЗОНАНТТЫК ЖҮРҮШҮН АНАЛИЗДӨӨ ЖАНА СИНТЕЗДӨӨ

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АНАЛИЗ И СИНТЕЗ РЕЗОНАНСНЫХ И АНТИРЕЗОНАНСНЫХ ПРОЦЕССОВ В ЛИНЕЙНЫХ LC-ЦЕПЯХ

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ANALYSIS AND SYNTHESIS OF RESONANCE AND ANTIRESONANCE PROCESSES OF LINEAR LC CIRCUITS

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Электр схемалары ар кандай кыймалдаткыч системалары үчүн зор мисалдар боло алат. Анткени алардагы ейген маанилер жана ейген векторлор өтө маанилүү физика математикалык кыймылдаткыч теорияларына ээ ошондой эле көптөгөн касиеттүү практикалык колдонмолордогу орду чоң [1][2]. Бул изилдөө ишинде Вайнштейн методу аркылуу негизги маселелерге жолдор изделип жыйынтык көздөлгөн. Натыйжада көп жактуу LC схемаларынын белгиленген жыштыктары атайын жолдор аркылуу синтезделип жана анализделип жыйынтыка чыгарылат[3][6].

Негизги сөздөр: *ейген мааниси, ортоңку маселелер, Вайнштейн методу контурдук матрица, индуктивдүүлүк контурдук матрица.*

Электрические схемы иллюстрируют колебательные системы с сосредоточенными параметрами. Метод Вайнштейн из промежуточных задач был изменен для системы с сосредоточенными параметрами – электрических цепей[1][2]. Отношения между собственными сопротивлений отдельных отраслей цепи и петли сопротивлений устанавливается. В результате многомерного LC цепи с заданными частотами будут синтезированы и проанализированы с помощью разработанных методов[3][6].

Ключевые слова: *собственное значение, промежуточные задачи, метод Вайнштейна, контурная матрица, контурная матрица индуктивности.*

Electric circuits exemplify oscillatory systems with concentrated parameters. The problem on their eigenvalues and eigenvectors is, on the one hand, one of the most important problems of mathematical physics and the oscillation theory, and on the other hand, universal, since it has many diverse applications[1][2]. Weinstein method of intermediate problems has been modified for a system with concentrated parameters – electrical circuits. Basis (initial) problem for this case of the intermediate problems method is defined. A relationship between eigenvalues (proper frequencies) of impedances of separate branches of the circuit and loop impedances is established. As a result, multi-dimensional LC circuits with predetermined frequencies will be synthesized and analyzed through developed techniques [3][6].

Key words: *eigen values, intermediate problems, Weinstein method, loop matrix, loop impedance.*

Introduction

Considering LC-circuit, consisting of n branches, in which each of them connected serially with LC-elements. A set of such not connected branches make up primitive circuit, whereas set of l_i and C_i (i=1,2,…,n) individual elements of the inductors and capacitors represented as diagonal matrices L^d and C_d, respectively. We denote Γ k×n loop matrix circuit, where k- is number of independent loops, then loop matrices of inductances L^k and capacitances defined by the following transformations [4]

$$L^k = \Gamma L^d \Gamma'$$

$$C_k = \Gamma C_d \Gamma'. \qquad (1)$$

In this case, k-order loop matrix of impedances is equal to

$$Z^k = \lambda L_k - C_k, \qquad (2)$$

Where λ- eigenvalues of the generalized problem (6.3) (or (6.4)), squares are equal to their own angular frequencies $\lambda = \omega^2$ of circuits.

L_k and C_k are the original matrix for solving the generalized problem of the eigenvalues, the solution of which can be carried out both by means of a method for solving the problem of the eigenvalues of the matrix $D = (C_k)^{-1^k}$. Without going into technical details relating to methods of solving the generalized problem of the eigenvalues, we will present the result of its decision by the following matrices: regular matrix V-order of k, whose columns are eigenvectors and diagonal matrix of eigenvalues of S, Then the elements of the transformed matrix of the loop impedances ZS^k will be equal to:

$$zs_{ij} = \begin{cases} \lambda - \lambda_i & i = j \\ 0 & i \neq j \end{cases}, \qquad (3)$$

Where E^k-identity matrix of order k

Afterwards is the result of simple transformations

$$ZS_k = V'Z^kV = V'(\lambda L_k - C_k)V = \lambda V'L_kV - V'C_kV = (\lambda - \lambda_k)E^k.$$

It makes it easy to draw Z_k. Indeed,

$$(ZS_k)^{-1} = V^{-1}(Z_k)^{-1}(V')^{-1}$$

From where

$(Z_k)^{-1} = V(ZS_k)^{-1}V'$,

Where $(ZS_k)^{-1}$-diagonal matrix, in which magnitudes$(\lambda - \lambda_i)^{-1}$ are located

Denoting that representation (5) of inverse matrix $(Z^k)^{-1}$ significantly makes easy to calculate for different values of $\lambda = \omega^2$ [5]

Analysis of resonance and antiresonance processes in linear LC circuits

Assume that, branches of LC circuits are supplied with EMF with phase angle of zero, which is denoted by E^b n- dimensional vector amplitude of periodic voltage in independent loops. They are obviously linked by the following transformation $E^k = \Gamma' E^b$. The latter one lets us represent the amplitude of the branch currents and loop currents as a function of frequency

$$i_k(\omega) = (Z_k)^{-1}E^k = V(ZS_k)^{-1}V'E^k,$$

Where $i_k(\omega)$ – k-dimensional vector amplitude of loop currents, the components of which are (3) function of frequency.

From (6) immediately determined n- dimensional vector amplitude of branch currents.

$$i_b(\omega) = \Gamma' i_k(\omega).$$

Using (6) and (7) cases, possible to calculate both squares of loop currents and branch currents without any difficulties, i.e. Obtaining representation of the power in the branches and in the loops, which are proportionally squares of these currents as a function of frequency. However, without giving break let us consider link between antiresonance effects with modified methods of intermediate problems

As it is known, antiresonance effects occur under certain excitations of independent circuits. In particular, if the vector amplitudes acting harmonic voltage takes the form

$$E^k=(0,0,..., E_i^k,\dots,0),$$

i.e. If the external excitation is different from zero only in i^th^ loop, then at certain frequencies, amplitude of current in this loop will be equal to zero, but in the other loops-different from zero. It is easy to determine the equation for these frequencies

Indeed, the solution of the equation

$$E^k = Z_k i_k$$

it can be represented as follows

$$i_k^i = \sum_{j=1}^{k} E_j^k \frac{\Delta_{ij}}{|Z_k|} = E_i^k \frac{\Delta_{ii}}{|Z_k|},$$

Where Δ_{ij} – associate member of z_{ij} matrix Z_k.

(5)

Representation of loop matrix of this circuit in the basis, associated with its eigenvectors as well as simple criterion conservatism of eigenvalues of primitive circuit which are proven in the previous chapters, makes easy to analyze and realize the synthesis of circuits with predefined values of resonance frequencies.

In this case i^{th} component of i_k^ivector loop currents i_k becomes equal to zero.

$$\Delta_{ii} = 0.$$

It is obviously that (10) has polynomial of order k-1, as far as equal to determinant of matrix k-1 order, obtained by cancellation from Z_k i-row and i- column. Solution of equation (10) determines vector θ_i, consisting of k-1 antiresonance frequencies of i^{th} loop [7][8]. (6)

Conclusion

The above results allow to set physical meaning of intermediate function of Weinstein applied to electric circuits.

Proposition 10. If the vector amplitude acting harmonic voltage, has the form of; (7)

$$E^k=(0,0,..., E_i^k,\dots,0),$$

where $E_i^k = 1$

Then intermediate function of Weinstein has function of current amplitude i^{th}-loop from frequency.

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