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DYNAMICS OF NUMBER OF SYSTEMS THE PHYTOPHAGE- ENTHOMOPHAGE

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ДИНАМИКА ЧИСЛЕННОСТИ СИСТЕМ ФИТОФАГ-ЭНТОМОФАГ

The observed violation of nature ecosystems stability and its disturbing often linked with anthropogenic influence. The population's magnitude dynamics analysis in ecosystems of insects, models, like phytophage – enthomophage, double phytophage - one enthomophage, are researching in this job. Phase portrait of outbreak gradation's different states are explained. We identify model parameters that lead to explosive dynamics. In particular, we demonstrate that the general model the phytophage- enthomophage opens possibility of a quantitative estimation of their interaction which can be used for forecasting of dynamics of number.

Key words : Ecosystem, Model phytophage – enthomophage, System a predator – two prey, Phase portraits, Variety and stability, Regulation.

Наблюдаемые нарушения стабильности природных экосистем и их разрушение часто связывают с антропогенным воздействием. В работе исследуется анализ динамики численности популяций насекомых в экосистеме, модели фитофаг – энтомофаг, два фитофага – один энтомофаг. Получены фазовые портреты различных режимов вспышек массовых размножений.

Introduction

Now observed violation of nature ecosystems stability and its disturbing often linked with anthropogenic influence and with influence of natural factors. The mankind searches various possibilities on maintenance of stability of ecosystems.

It is necessary to notice that the actions directed on increase of biocenose stability, should be based on detailed knowledge of regulation mechanisms and interaction character of its components. Therefore the role of studying various factors increases, for example: biotic and abiotic factors of maintenance a specific variety and stability and ways of communities structure regulation, forecasting of consequences introduction and-or elimination kinds from an ecosystem.

At the analysis of number dynamics it is important to receive an overall interaction picture of populations of insects in an ecosystem. It can be reached at research of phase portraits of phytophages at interaction with enthomophage.

Let's consider systems a phytophage – enthomophage, double phytophage - one enthomophage. The system of model phytophage – enthomophage [1- 9]:

$$\begin{aligned} \frac{dx}{dt} &= px(1-\beta x-\frac{z}{1+x^2}) \\ \frac{dz}{dt} &= cz(-\alpha - z+\frac{\gamma x^2}{1+x^2}) \end{aligned}, \qquad (1)$$

where $x(t)$, $z(t)$ - number accordingly preys and predators in system at the moment of time t, $\alpha,\beta,\gamma,p,c=const\geq 0$;

px - growth rate of population of prey;

$p\beta x^2$ - speed of destruction of the individuals, caused by action inside population mechanisms;

$\frac{pxz}{1+x^2}$ - speed of destruction of prey by predators at sufficient small value x;

αcz - speed of natural death rate of predators;

cz^2 - speed of death rate of population of the predators, caused by action inside population a competition;

$\frac{c\gamma zx^2}{1+x^2}$ - speed of a gain of number of predators at the expense of eating of victims.

Let's consider behavior of the main things an isoclinic lines of system (1). On co-ordinate axes which are integrated lines (1), we have following special points: (0, 0) and ($1/\beta$, 0). The point (0,0) always is a saddle with entering separations axis Z and leaving separations an axis X. The point ($1/\beta$, 0) will be stable knot, if

$F = -\alpha + \dfrac{\gamma}{1+\beta^2} < 0$ in this case population of predators degenerates at any final initial values of number population or will be Saddle with entering separations axis X at performance a condition F>0 and then in system (1) realized only modes of joint coexistence of both populations.

If parameters lie on bifurcation surfaces F=0, the point ($1/\beta$, 0) is steady a saddle - knot with central sector in the first fourth quarter.

For definition of number of stationary points in $\operatorname{int} R_2^+$ we will consider the following system of the algebraic equations:

$$\begin{cases} 1-\beta x - \dfrac{z}{1+x^2} = 0 \\ \alpha + z - \gamma \dfrac{x^2}{1+x^2} = 0 \end{cases} \qquad (2)$$

Whence, excepting unknown z, we receive:

$$-\beta x^5 + x^4 - 2\beta x^3 + (2+\alpha-\gamma)x^2 - \beta x + (1+\alpha) = 0 \qquad (3)$$

For existence of 5 roots with a positive valid part it is necessary, that conditions were simultaneously satisfied $T_1T_2 > 0$, $T_2T_3 < 0$ и $T_3T_4 > 0$. i.e. that $\gamma - \alpha > 0, [(1+\alpha-\gamma)^2 - (\alpha+1)] < 0$ and $[(1+\alpha-\gamma)^2 - (\alpha+1)] > 0$, That, obviously, it is impossible, hence, there are only three changes of a sign and there can not be five roots with a positive valid part at a polynomial (3). Thus, from a positive part of phase space of system (2) there can be no more than three stationary points, as well as in nonparametric model of system a phytophage- enthomophage [2-4]. In figures 1-4 results of integration of system are presented (1) method of Runge-Kutt at various values of parameters. Lines with arrows – system trajectories, without arrows – isoclinic lines of vertical and horizontal inclinations. Apparently from these illustrations, all most important modes [1-4] are realized in (1) and it can be quite used for restoring of dynamics modes of number populations on experimental data. The interest represents the analysis of cases $f = x^3$ and $f = p_1x + p_2x^2$. Last possesses one property that at $p_1 = 0$ it passes in (1), at $p_2 = 0$ in model Lotka - Volterra with saturation [5,6], and at $p_1 = 0, p_2 \square 0$ in classical model Lotka – Volterra [1-4,7]. Such model will be more flexible at work with experimental a material as possesses wider spectrum of dynamic modes. Model (1) in this case appears as its special case.

Phase portraits:

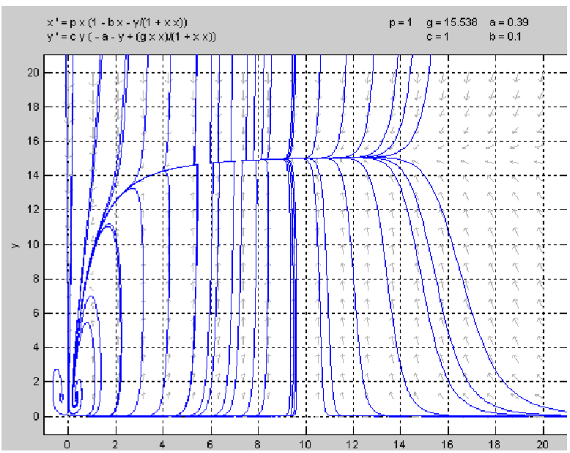


Figure 1. A phase portrait of system (1), at $p = 1, \alpha = 0.39, \beta = 0.1, \gamma = 15.538, c = 1$ a mode of the fixed flash

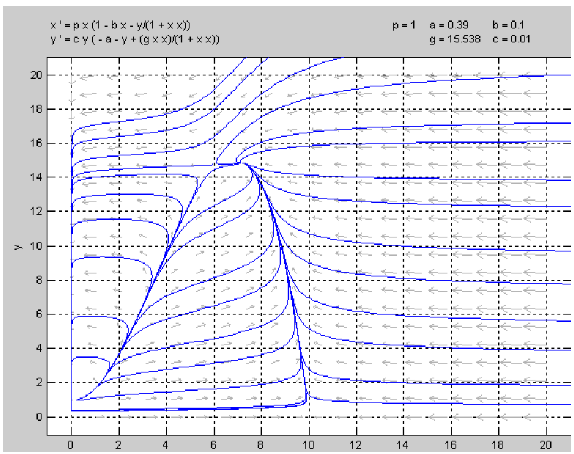


Figure 2. A phase portrait of system (1), at $p = 1, \alpha = 0.39, \beta = 0.1, \gamma = 15.538, c = 0.01$ a mode of the reversive flash

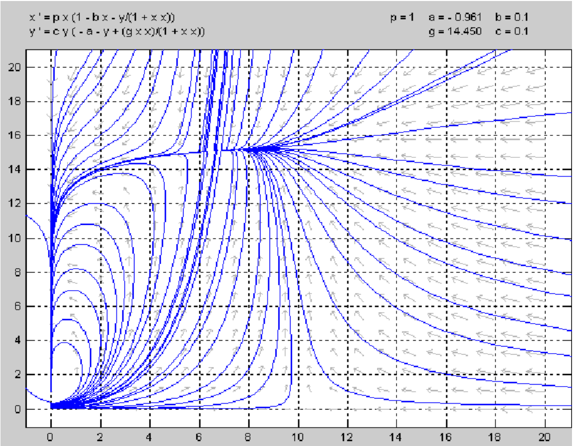


Figure 7. A phase portrait of system (1), at $p=1.0, \alpha=-0.961, \beta=0.1, \gamma=14.450, c=0.1$ a mode of the fixed flash

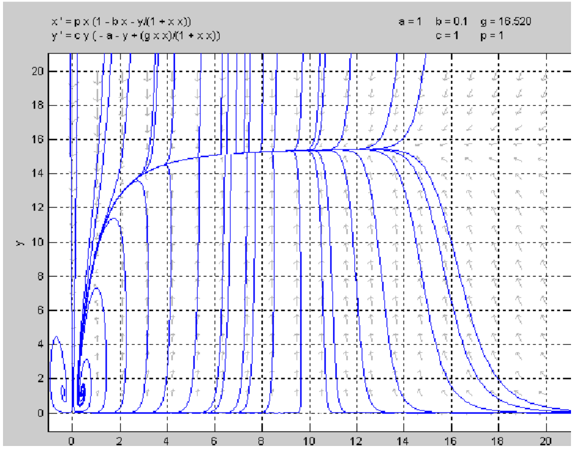


Figure 8. A phase portrait of system (1), at $p=1.0, \alpha=1, \beta=0.1, \gamma=16.520, c=1$ a mode of the fixed flash

How influences introduction a new kind of a phytophage (an additional forage for a predator) stability of an ecosystem?

It is necessary to enter into the phytophage system, not capable to give flash of mass reproduction.

For this purpose we will consider system a predator – two prey, which a prey can among themselves do not compete:

$$\frac{dx}{dt}=\lambda x(\alpha_1-\beta_1 x-\frac{\gamma_1 z}{1+x^2})$$
$$\frac{dy}{dt}=\mu y(\alpha_2-y-\gamma_2 z) \tag{4}$$
$$\frac{dz}{dt}=z(-\alpha_3-z+\frac{\gamma_3 x^2}{1+x^2}+\gamma_4 y)$$

System of the nonlinear algebraic equations:

$$z=\frac{(\alpha_1-\beta_1 x)(1+x^2)}{\gamma_1}=\frac{\alpha_2-y}{\gamma_2}=\frac{\gamma_3 x^2}{1+x^2}+\gamma_4 y-\alpha_3 \tag{5}$$

allows to define co-ordinates of all stationary points of system (5). Excluding from system (6) variables z and y, we receive a polynomial of the fifth degree concerning a variable x:

$$x^5-\frac{\alpha_1}{\beta_1}x^4+2x^3+x^2\left[\frac{\gamma_1\gamma_3+\alpha_2\gamma_1\gamma_4-\alpha_3\gamma_1-2\alpha_1-2\alpha_1\gamma_2\gamma_4}{\beta_1(1+\gamma_1\gamma_4)}\right]+x+\frac{\alpha_2\gamma_1\gamma_4-\alpha_3\gamma_1-\alpha_1-\alpha_1\gamma_1\gamma_4}{\beta_1(1+\gamma_1\gamma_4)}=0 \tag{6}$$

For a finding of co-ordinates of points of balance in $\operatorname{int} R_3^+$.

From system (6) it is easy to receive a similar polynomial for a finding of points of the balance lying in $\operatorname{int}(X\circ Z)$, i.e. on a plane where the effect "escape" can be realized:

$$x^5-\frac{\alpha_1}{\beta_1}x^4+2x^3+x^2\frac{\gamma_1\gamma_3-\gamma_1\alpha_3-2\alpha_1}{\beta_1}+x-\frac{\alpha_1+\alpha_3\gamma_1}{\beta_1}=0 \tag{7}$$

Selecting parameters of polynomials (6) and (7), it is possible to construct such phase space in $\operatorname{int} R_3^+$ systems (5) at which will be three points lying in $\operatorname{int}(X\circ Z)$, and one point in $\operatorname{int} R_3^+$. Really, let $\alpha_1=10, \gamma_1=15, \alpha_3=1, \gamma_3=10$. Then the number of points in $\operatorname{int}(X\circ Z)$ depends on number of positive roots of a polynomial (7) which in this case looks like.

$$x^5-10x^4+2x^3+115x^2+x-25=0 \tag{8}$$

On an interval (0,10). The polynomial (8) has such three roots. The number of points in is defined thus by a polynomial (6):

$$\tilde{P}(x) = x^5 - 10x^4 + 2x^3 + x^2\frac{115 + 15\alpha_2\gamma_4 - 20\gamma_2\gamma_4}{1+\gamma_2\gamma_4} + x - \frac{115\alpha_2\gamma_4 - 35 - 10\gamma_2\gamma_4}{1-\gamma_2\gamma_4} = 0 \qquad (9)$$

Let's designate through K_1 and K_2 following sizes:

$$K_1 = \frac{115 + 15\alpha_2\gamma_4 - 20\gamma_2\gamma_4}{1+\gamma_2\gamma_4}, K_2 = -\frac{115\alpha_2\gamma_4 - 35 - 10\gamma_2\gamma_4}{1+\gamma_2\gamma_4}$$

It is obvious that for any fixed positive values of parameters γ_2 and γ_4, it is possible to choose parameter α_2 such that the polynomial (9) will have one root on an interval (0,10).

If the parameter α_2 such that is carried out an inequality $K_1 \geq 10^3$ then $10x^4 < K_1x^2$ on an interval (0,10) and, hence, the polynomial $\tilde{P}$ monotonously increases on this interval.

As size $K_2 < 0$ at enough great values of parameter $\alpha_2, \tilde{P}(0) < 0$ and $\tilde{P}(10) > 0$ a polynomial (8) has exactly one root.

Thus, system of the ordinary differential equations (a model special case (4)).

$$\begin{aligned}
\frac{dx}{dt} &= \lambda x(10 - x - \frac{15z}{1+x^2}) = \lambda x Q_1 \\
\frac{dy}{dt} &= \mu y(\alpha_2 - y - \gamma_2 z) = \mu y Q_2 \\
\frac{dz}{dt} &= \varepsilon z(-1 - z + \frac{10x^2}{1+x^2} + \gamma_4 y) = \varepsilon z Q_3
\end{aligned} \qquad (10)$$

At certain values of parameters α_2, γ_2 and γ_4 also will have one equilibrium state of century $\operatorname{int} R_3^+$. Let's consider stationary conditions of system of the equations (10).

The point (0,0,0) is a saddle, with one target line - an axis z. The Jacobean (10) the following systems:

$$J = \begin{Vmatrix} \lambda Q_1 + \lambda x(-1 - \frac{30xz}{(1+x^2)^2}) & 0 & -\frac{15\lambda x}{1+x^2} \\ 0 & \mu Q_2 - \mu y & -\mu\gamma_2 y \\ \frac{20\varepsilon xz}{(1+x^2)^2} & \varepsilon\gamma_4 z & \varepsilon Q_3 - \varepsilon z \end{Vmatrix}$$

Let $(\bar{x}, \bar{y}, \bar{z})$ - a unique steady state of system in $\operatorname{int} R_3^+$. The Jacobean in this point looks like:

$$J = \begin{Vmatrix} -\lambda \bar{x} - 30\lambda\frac{\bar{x}^2\bar{z}}{(1+\bar{x}^2)^2} & 0 & -\lambda\bar{x}\frac{15}{1+x^2} \\ 0 & -\mu\bar{y} & -\mu\gamma_2\bar{y} \\ 20\varepsilon\frac{\bar{x}\bar{z}}{(1+x^2)^2} & \varepsilon\gamma_4\bar{z} & -\varepsilon\bar{z} \end{Vmatrix}$$

Jacobean own number’s satisfy to a following cubic equation:

$$-\xi^3 + A_1\xi^2 + A_2\xi + A_3 = 0. \qquad (11)$$

Where coefficients A_k depend on system parameters:

$$A_1 = -\lambda \bar{x} - 30\lambda \frac{\bar{x}^2 \bar{z}}{(1+\bar{x}^2)^2} - \mu \bar{y} - \varepsilon \bar{z} < 0,$$

$$A_2 = -\frac{300\lambda\varepsilon \bar{x}^2 \bar{z}}{(1+\bar{x}^2)^3} - \bar{y}\bar{z}\varepsilon\mu(1+\gamma_2\gamma_4) - (\lambda \bar{x} + 30\lambda \frac{\bar{x}^2 \bar{z}}{(1+\bar{x}^2)^2}(\mu \bar{y} + \varepsilon \bar{z})) < 0,$$

$$A_3 = -(\lambda \bar{x} + 30\lambda \frac{\bar{x}^2 \bar{z}}{(1+\bar{x}^2)^2})(\bar{y}\varepsilon\mu \bar{z} + \bar{y}\bar{z}\varepsilon\mu\gamma_2\gamma_4) - 300\lambda\mu\varepsilon \frac{\bar{x}^2 \bar{z}^2}{(1+\bar{x}^2)^3} < 0$$

Hence, the polynomial (11) has no positive roots and under the theorem of signs on Descartes has either one, or three negative real roots.

As for a polynomial (11) conditions of criterion of Raus - Gurvits are executing, so the steady state $(\bar{x}, \bar{y}, \bar{z})$ is satisfy.

Thus, simultaneous existence of three kinds can exclude modes of mass flashes.

Thus degeneration of one population $(y \equiv 0)$ leads to that another has an opportunity to escape from under the control of enemies.

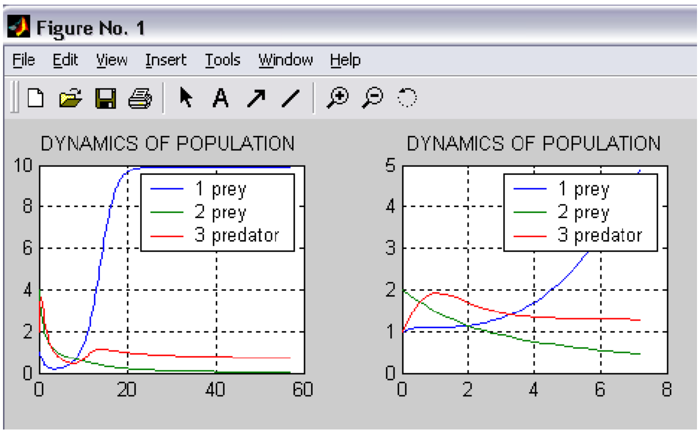
Here we see possibility of management population by means of release in system additional view phytophage.

The most important in this direction is researches of mathematical models of system the predator-two (or more) prey, each of which is capable to pass to mass reproduction.

Similar models should explain mechanisms and stages of development of the interfaced centers which are often observed in actual practice.

Portraits of dynamics population of model (4):

Parametres
$\lambda = 0.61,$
$\alpha_1 = 1,$
$\beta_1 = 0.1,$
$\gamma_1 = 1$
$\mu = 0.1,$
$\alpha_2 = 0.4,$
$\gamma_2 = 1$
$\alpha_3 = 0.8,$
$\gamma_3 = 1.5,$
$\gamma_4 = 1.3$



Conclusion

The forecast based on modeling decisions will be more exact as instead of probability of transition from area in area we will have all trajectory of change of number, and more long-term for the same reasons.

But development of this method is interfaced to necessity of overcoming of some barriers among which one of the cores is the choice of an obvious kind of laws of self-control and interaction.

The error in a choice of laws brings to nothing the subsequent decision of a problem.

The criteria, allowing to carry out a choice of laws, now is not present.

And actually while a unique way of the decision of these problems is search of the mathematical models, conditions satisfying to a complex that provides realization in model of a necessary spectrum of dynamic modes.

From these positions the model analysis (1,4), resulted in the present work, is represented to us the important step to the problem decision.

Reception on models of necessary sets of dynamic modes creates certain conveniences at work with experimental data.

However researches of more difficult models which can possess the big set of modes that will allow describing natural ecosystems more adequately can be demanded.

Heuristic value of the analysis of the considered models remains for more wide range of multicomponent systems.

Simultaneously within the limits of the general model the phytophage- enthomophage opens possibility of a quantitative estimation of their interaction which can be used for forecasting of dynamics of number.

Appendix

Phytophage - an animal eating vegetative food.

Enthomophage - an organism eating insects. Enthomophage can be an insectivorous vertebrate animal, a predator from the world invertebrate, a parasite, a predatory plant, etc.

Fixed flash is caused by inability enthomophages to return system in a stable condition. The similar type of flash is usually observed in artificial bio and agrocenoses.

Permanent flash is caused by inability enthomophages to fix number of phytophages in a stable and metastable condition. Functioning of such systems depends on original influence of phytophages on a fodder plant.

Reversive flash is interfaced to loss regulated properties of system towards limiting rarefy populations of phytophages without fixing of this condition.

Actually flash of mass reproduction is connected with loss regulated properties of system towards limiting growth of number of population of phytophages without fixing of this condition.

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