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FREE LINEAR OSCILLATIONS OF THE PENDULUM SEISMIC MODELING

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СВОБОДНЫЕ ЛИНЕЙНЫЕ КОЛЕБАНИЯ МАЯТНИКА СЕЙСМИЧЕСКОГО МОДЕЛИРОВАНИЯ

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В этой статье рассмотрены исследования свободных линейных колебаний маятника сейсмической модели с достаточно низкой скоростью в достаточно малой области пространства вокруг его стабильного положения равновесия (θ = ψ = φ = 0). Исследование было проведено в модифицированных координатах Эйлера. Уравнения движения и другие отношения вне сил считаются равными нулю,члены второго и более высокого порядка пренебречь.

In this article study of free linear oscillations of the pendulum seismic model with a sufficiently low speed in a sufficiently small spatial region around its stable equilibrium position (θ = ψ = φ = 0). The study was conducted in the modified Euler coordinates. The equations of motion and other relationships outside forces assumed to be zero, deterred members of the first order, the members of the second and higher order neglected.

Hamilton's equations to have a symmetric pendulum in the modified coordinate system, according to [1], will be as follows:

$$\dot{\psi} = \frac{\partial H^*}{\partial P_\psi^*},\ \dot{\theta} = \frac{\partial H^*}{\partial P_\theta^*},\ \dot{\varphi} = \frac{\partial H^*}{\partial P_\varphi^*},\ \dot{P}_\psi^* = -\frac{\partial H^*}{\partial \psi} + Q_\psi,$$

$$\dot{P}_\theta^* = -\frac{\partial H^*}{\partial \theta} + Q_\theta,\ \dot{P}_\varphi^* = -\frac{\partial H^*}{\partial \varphi} + Q_\varphi,\quad (1)$$

where the Hamiltonian H * expressed in terms of the momenta of the natural system as:

$$H^* = \frac{1}{2}\left\{\frac{A_2\left(P_\psi^* + P_\varphi^*\sin\theta\right)}{J_x J_y \cos^2\theta} + \frac{\left(J_y - J_x\right)P_\theta^*\left(P_\psi^* + P_\varphi^*\sin\theta\right)\sin 2\varphi}{J_x J_y \cos\theta} + \right.$$
$$\left. + \frac{A_1 P_\theta^{*2}}{J_x J_y} + \frac{P_\varphi^{*2}}{J_z}\right\} + \left\{\frac{c\varphi^2}{2} + ML_0 g\left(1 - \cos\theta\cos\psi\right) - ML_0 w_Z\right\}\quad (2)$$

Here P_ψ^*, P_θ^*, P_φ^* – the generalized momenta of the natural system

$$P_\psi^* = J_x\omega_x\gamma_1 + J_y\omega_y\gamma_2 + J_z\omega_z\gamma_3,$$

$$P_\theta^* = J_x\omega_x\cos\varphi - J_y\omega_y\sin\varphi,\quad P_\varphi^* = J_z\omega_z,\quad (3)$$

generalized forces and can be represented as:

$$Q_\psi = -kP_\psi^* + c\varphi\sin\theta + Q_\psi',\ Q_\theta = -kP_\psi^* + Q_\theta',$$
$$Q_\varphi = -kP_\varphi^* + Q_\varphi'\quad \theta_\psi' = F_x\gamma_1 + F_y\gamma_2 + F_z\gamma_3,$$
$$\theta_\theta' = F_x\cos\varphi - F_y\sin\varphi,\ \theta_\varphi' = F_z.$$

F_ψ, F_θ, F_φ defined by the relations (4)

$$F_x = -\left[\varepsilon_x J_x + \left(J_z - J_y\right)\Omega_y\Omega_z + 2\left[\Omega_y\omega_z J_{(zx)} - \Omega_z\omega_y J_{(xy)}\right]\right],$$
$$F_y = -\left[\varepsilon_y J_y + \left(J_x - J_z\right)\Omega_x\Omega_z + 2\left[\Omega_z\omega_x J_{(xy)} - \Omega_x\omega_z J_{(yz)}\right]\right],$$
$$F_z = -\left[\varepsilon_z J_z + \left(J_y - J_x\right)\Omega_x\Omega_y + 2\left[\Omega_x\omega_y J_{(yz)} - \Omega_y\omega_x J_{(zx)}\right]\right]\quad (4)$$

Investigate the free linear oscillations of the pendulum seismic model with a sufficiently low speed in a sufficiently small spatial region around its stable equilibrium position (θ = ψ = φ = 0) [1,2]. The study will be carried out in the modified Euler coordinates. The equations of motion and other relations (1) – (4) we put the external force equal to zero, and retain members of the first order, neglecting terms of second and higher order. From these relations we eliminate the generalized momenta.

As a result, we obtain a system of equations of the second order with respect to the angular coordinates:

$$\ddot{\theta} + 2k_0\dot{\theta} + n_1^2\theta = 0,\ \ddot{\psi} + 2k_0\dot{\psi} + n_2^2\psi = 0,$$
$$\ddot{\varphi} + 2k_0\dot{\varphi} + n_3^2\varphi = 0.\quad (5)$$

where $k_0 = \frac{k}{2}$ – damping factor, n_1, n_2, n_3 – own cyclic oscillation frequency, with:

$$n_1 = \sqrt{\frac{ML_0 g}{J_x}},\quad n_2 = \sqrt{\frac{ML_0 g}{J_y}},\ n_3 = \sqrt{\frac{C}{J_z}}.\quad (6)$$

As can be seen from (5) the angular coordinate system (ψ, θ, φ) is the main. Each main coordinate is described by its equation independently of the other angular coordinates. In the linear approximation, the exchange of energy between the degrees of freedom occurs.

Consider own sustained oscillations, which are described by the system of equations:

$$\ddot{\theta} + n_1^2\theta = 0;\ \ddot{\psi} + n_2^2\psi = 0;\ \ddot{\varphi} + n_3^2\varphi = 0.\quad (7)$$

Relations (6) are well-known expressions for the vertical and torsional oscillations of the pendulum. For symmetric pendulum $n_1 = n_2 = n$. Using data about the model pendulum (pendulum, located at the station "Medeo" [3]), we estimate the natural frequencies and periods of linear oscillations of the pendulum:

$$n_1 \approx n_2 = 2{,}56\,Hz,\ n_3 = 19{,}48\,mHz,$$

$$T_1 \approx T_2 = \frac{2\pi}{n_1} = 2{,}45\,sec,\ T_3 = \frac{2\pi}{n_3} = 5{,}375\,min.$$

That is the order of the cyclic frequency of precession and nutation oscillations of hertz, and torsional vibration - millihertz. Accordingly, the periods of precession and nutation oscillations are seconds and torsion - tens of minutes, $\frac{n_3}{n_1} = 7{,}6 \cdot 10^{-3}$ «1.

Note that the oscillations described by the system of equations (7), are harmonic oscillations. The solution of equations (7), using the initial conditions can be written as:

$$\theta(t) = A_1 \cos(n_1 t + e_1), \quad \psi(t) = A_2 \cos(n_2 t + e_2),$$
$$\varphi(t) = A_3 \cos(n_3 t + e_3), \tag{8}$$

where the amplitudes A_1, A_2, A_3, and the initial phases of e_1, e_2, $e_3$are defined as:

$$A_1 = \sqrt{\theta_0^2 + \left(\frac{\dot{\theta}_0}{n_1}\right)^2}, \quad A_2 = \sqrt{\psi_0^2 + \left(\frac{\dot{\psi}_0}{n_2}\right)^2},$$
$$A_3 = \sqrt{\varphi_0^2 + \left(\frac{\dot{\varphi}_0}{n_3}\right)^2},$$
$$\mathrm{tg}\, e_1 = -\frac{\dot{\theta}_0}{n_1\theta_0}, \quad \mathrm{tg}\, e_2 = -\frac{\dot{\psi}_0}{n_2\psi_0}, \quad \mathrm{tg}\, e_3 = -\frac{\dot{\varphi}_0}{n_3\theta_0}. \tag{9}$$

Trajectory, which describes the projection of the center of mass of the pendulum on a horizontal plane (oξζ) describes the relations [1,2]:

$$\xi_{\tilde{N}} = -L_0\psi = -A_2 L_0 \cos(n_2 t + e_2),$$
$$\zeta_{\tilde{N}} = -L_0\theta = A_1 L_0 \cos(n_1 t + e_1). \tag{10}$$

Consequently, the pendulum moves so that the projection of its center of mass on the horizontal plane describes a Lissajous curve, and it performs torsional oscillation around the axis OZ. If the frequency ratio n_1/n_2 is a rational number, then the Lissajous figures - closed curves; if not, the Lissajous curves - make, and the pendulum will never return to the original point. If $n_1= n_2$ (symmetric pendulum), the Lissajous figures are in the shape of an ellipse, and the projection onto the horizontal plane of oscillation will be elliptically polarized. If $A_1=A_2$, the oscillations will be circularly polarized. If the phase difference $e_2-e_1=(2k+1)\pi/2$ ($k = 0, \pm1, \pm2, \ldots$), the ellipse degenerates into a line segment, and the vibrations are plane-polarized (linearly polarized), and the pendulum will oscillate a fixed vertical plane and twisted around its axis. Since the design of the pendulum such that $J_x \approx J_y$ and therefore, $n_1 \approx n_2$, then the oscillations are close to elliptically polarized, but the path will, in general, not closed. Fluctuations are linearly polarized for a symmetric pendulum (see. (10)), when the initial conditions are related by $\frac{\dot{\theta}_0}{\theta_0} = \frac{\dot{\psi}_0}{\psi_0}$

Multiply the first equation (7) by , the second – on $\dot{\psi}$, the third – on $\dot{\varphi}$. Due to the current and integrate over time. Obtain the law of conservation of the total energy E of a conservative system:

$$E = \frac{1}{2}\left(J_x \dot{\theta}^2 + J_y \dot{\psi}^2 + J_z \dot{\varphi}^2\right) +$$
$$+\frac{1}{2}\left(ML_0 g\left(\theta^2 + \psi^2\right) + c\varphi^2\right) = const, \tag{11}$$

where the first term in parentheses – the kinetic energy, the second – the potential energy of the system, which consists of a potential energy due to gravity and the force of elasticity of the filament. Moreover, it will be the law of conservation of the total energies of nutation, precession and rotational motions:

$$E_\theta = \frac{1}{2}\left(J_x \dot{\theta}^2 + ML_0 g\theta^2\right) = C_\theta = const,$$
$$E_\psi = \frac{1}{2}\left(J_y \dot{\psi}^2 + ML_0 g\psi^2\right) = C_\psi = const, \tag{12}$$
$$E_\varphi = \frac{1}{2}\left(J_z \dot{\varphi}^2 + C\varphi^2\right) = C_\varphi = const.$$

Note that for a symmetric pendulum when $A_1 = A_2$, $e_1 - e_2 = (2k+1)\pi/2$, (6) it follows that $\psi^2 + \theta^2 = const$, and then the cosine of the angle of the thread of the pendulum with the vertical axis βзоηconstant. Consequently, the pendulum in this case makes a conical motion and twists around the axis OZ.

Let us pass to the consideration of the damped oscillations of the pendulum, to which we turn to the equations (5). Consider, for example, the first equation (5). For the rest of equations (5), the results are similar. Solutionofthefirstequation (5) intheform:

$$\theta = Ce^{\lambda t}. \tag{13}$$

Substituting (13) into the first equation (5), we obtain the characteristic equation:

$$\lambda^2 + 2k_0\lambda + n_1^2 = 0, \tag{14}$$

whose decisions:

$$\lambda_{1,2} = -k \pm ik_1, \; k_1 = \sqrt{n_1^2 - k_0^2}, \tag{15}$$

with the proviso that $n_1>k_0$. The real part of λ is the damping factor, and the imaginary part of λ is the natural frequency of oscillation, with $k_1<n_1$. The value of the natural frequency of the damped oscillations $k_1$decreased due to the forces of friction.

When $n_1>k_0$solution of the first equation (5) can be written as:

$$\theta(t) = A_1 e^{-k_0 t} \cdot \cos(k_1 t + e_1), \qquad (16)$$

where the amplitude A_1 and phase shift are determined from the initial conditions:

$$A_1 = \sqrt{\theta_0^2 + \left(\frac{\dot{\theta}_0 + k_0\theta_0}{k_1}\right)^2}, \ \mathrm{tg}\, e_1 = -\frac{\dot{\theta}_0 + k_0\theta_0}{\theta_0 k_1}. \qquad (17)$$

Conditional period T_1, the logarithmic decrement $q_1$and the relaxation time τ of damped oscillations are defined as:

$$T_1 = \frac{2\pi}{k_1}, \ q_1 = \ln\left[\frac{A_1(t)}{A_1(t+T_1)}\right] = k_0 T_1 = \frac{1}{N_1},$$

$$\tau_1 = N_1 T_1 = \frac{1}{k_0}, \qquad (18)$$

where N_1– number of oscillations, after which the amplitude is reduced by a factor e, and τ_1– time required for this. When $n_1<k_0$ coefficients λ_1, λ_2 are real and negative, so that the motion will be aperiodic with two damping coefficients. The rate of change of energy, similar to (11), (12), for the damped oscillations take the form:

$$E_\theta = \frac{1}{2}\left(J_x \dot{\theta}^2 + ML_0 g\theta^2\right) = C_\theta = const,$$

$$E_\psi = \frac{1}{2}\left(J_y \dot{\psi}^2 + ML_0 g\psi^2\right) = C_\psi = const, \qquad (19)$$

$$E_\varphi = \frac{1}{2}\left(J_z \dot{\varphi}^2 + C\varphi^2\right) = C_\varphi = const.$$

Where Φ, Φ_θ, Φ_ψ, Φ_φ – power dissipative forces and the corresponding power dissipative forces on the angular coordinates. Note that the exchange of energy between the degrees of freedom for the damped oscillations of the linear system does not occur.

We estimate the conditional oscillation periods T_1, T_2, T_3, logarithmic decrement q_1, q_2, q_3, the relaxation oscillation $\tau_1 = \tau_2 = \tau_3 = \tau$ for the model of the pendulum by the damping coefficient $k_0=0,2$MHz, which belongs to the practical range of operating seismic pendulum [4], leaving the other parameters the same pendulum. Relaxation time for all the oscillations is the same:

$$\tau = \frac{1}{k_0} = 1 \text{ hour } 23 \text{ min } 33 \text{ sec};$$

the other parameters have the following numerical values:

$$T_1 \approx T_2 \approx 2{,}45\text{sec}, \ T_3 = 5{,}375 \text{ min},$$

$$N_1 = N_2 = 2041, \ N_3 = 16,$$

$$q_1 \approx q_2 = 0{,}00049, \ q_3 = 0{,}0625.$$

As can be seen from the above estimates, conditional periodsT_1, T_2, T_3, do not differ from the corresponding periods of sustained oscillations, due to the smallness of the damping coefficient. Logarithmic decrement for torsional vibrations about 127,5 times higher than the corresponding logarithmic decrement of nutation and precession oscillations. It takes approximately 2041 oscillation angle of nutation and precession, and only 164 of torsional vibrations to the corresponding amplitudes decreased by a factor e.

Relations (10) for the damped oscillations can be written as:

$$\xi_{\tilde{N}} = -L_0\psi = -A_2 L_0 e^{-k_0 t}\cos(n_2 t + e_2),$$

$$\zeta_C = L_0\theta = A_1 L_0 e^{-k_0 t}\cos(n_1 t + e_3). \qquad (20)$$

Therefore, the projection of the center of mass of the pendulum on a horizontal plane (οξζ) will have to describe not closed, and tapered curves Lissajous that after a sufficiently long time, contracted to a point, and the pendulum stops. In particular, the conical pendulum swings will occur so that the nutation angle will decrease with time, and the projection of the center of mass C will describe the tapered logarithmic spiral, until the pendulum stops. The damped oscillations will be plane-polarized, if $e_2 – e_1 = (2k+1)\pi/2$ ($k = 0, \pm1, \pm2, \ldots$), or if initial angular momentum about the vertical axis is equal to zero [1].

In the linear theory the principle of superposition; natural frequencies and damping coefficient do not depend on the initial conditions. Moreover, such variations have the property isochronism which consists in that, at zero initial velocity time during which the system moves from its initial position to its equilibrium position, regardless of the initial deflection.

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