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INIVERSE PROBLEM FOR THE TIME-DEPENDENT EQUATION BOLTZMANN TYPE

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ПРОБЛЕМА INIVERSE ДЛЯ ЗАВИСЯЩЕГО ОТ ВРЕМЕНИ УРАВНЕНИЯ ТИПА БОЛЬЦМАНА

Introduction and statement of the problem

The inverse problems where is required to determine some causative characterizations are the most wide sphere of application of ill posed problems theory. We have need of them when demanding characterizations are inaccessible for the direct observations. Thus working out the effective methods for the inverse problems is of great interest at present.

In this paper we study the inverse problem for the non-linear integro-differential kinetic equation. The direct problems were considered in our article [6]. Problems of this type appear in a number of distinct scientific areas; e.g., in the study of "runaway" electrons in fully ionized plasmas [2-3], in semiconductor theory [5], in the calculation of D.C. conductivity in biological membranes [4], in the behavior of a population of charged particles under the influence of a spatially uniform D.C. electric areas [1].

The inverse problem for the non-linear integro-differential kinetic equation

Let us consider the simplified non-linear kinetic equation

$$\frac{\partial f}{\partial t}+a(\upsilon)\frac{\partial f}{\partial \upsilon}+h(\upsilon)f(\upsilon,t)=\int_{-\infty}^{\infty}\kappa(\upsilon,\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',t))d\upsilon'+b(t)F(\upsilon,t)\equiv Kf\,, \tag{1}$$

$$(\upsilon,t)\in\Omega\equiv R\times R_+,\ R_+=[0,\infty),$$

under the initial condition

$$f(\upsilon,t)\big|_{t=0}=f_0(\upsilon),\quad \forall\upsilon\in R, \tag{2}$$

with the additional information

$$f(\upsilon,t)\big|_{\upsilon=\upsilon_0}=\psi_0(t),\quad \upsilon_0\in R,\quad t\geq 0,\ \psi_0(t)\in C^1(R_+) \tag{3}$$

and co-ordination condition

$$f_0(\upsilon_0)=\psi_0(0). \tag{4}$$

Here $F(\upsilon,t)$, $f_0(\upsilon)$, $F_0(\upsilon',f(\upsilon',t))$, $a(\upsilon)>0$, $k(\upsilon,\upsilon')\geq 0$, $\psi_0(t)$, $h(\upsilon)>0$ - are known functions, where $\int_{-\infty}^{+\infty}h(\upsilon)d\upsilon<+\infty$, $\int_{-\infty}^{+\infty}k(\upsilon,\upsilon')d\upsilon'=1$, $\int_{-\infty}^{+\infty}k(\upsilon,\upsilon')h(\upsilon')\left|F_{0f}(\upsilon',f)\right|d\upsilon'<+\infty$.

The problems type (1) – (4) describe the time evolution of the distribution function of particles in a phase of functions of velocity υ, or the time evolution of the specific intensity $f(\upsilon,t)$ of unpolarized light in a phase space of functions of frequency $h(\upsilon)$, given the initial distribution $f_0(\upsilon)$; the function $b(t)F(\upsilon,t)$ describes internal sources, while we are interested in determination the coefficient $b(t)$; the term containing $a(\upsilon)$ accounts for the effect of external forces.

The inverse problem is contained in determination pair of functions $(f(\upsilon,t);b(t))$, answered (1) - (4).

Thus let us consider the problem (1) - (4). The simplified direct problems of this type were solved by various methods, for example, semi group theory [7], where $b(t)\equiv 0$, $F_0(\upsilon, f(\upsilon,t))\equiv f(\upsilon,t)$, $a(\upsilon)=const$. We use the integral conversion

$$f(\upsilon,t)=Q(\upsilon,t)\exp\left(-\int_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right),\ \forall\upsilon\in R,\ \forall t\in R_+. \qquad (5)$$

As the result we obtain the problem :

$$\begin{cases}\dfrac{\partial Q(\upsilon,t)}{\partial t}+a(\upsilon)\dfrac{\partial Q(\upsilon,t)}{\partial\upsilon}=\exp\left(\int_{-\infty}^{\upsilon}\dfrac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty}k(\upsilon,\upsilon')h(\upsilon')F_0(\upsilon',f)d\upsilon'+b(t)F(\upsilon,t)\exp\left(\int_{-\infty}^{\upsilon}\dfrac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right), & (6)\\ Q(\upsilon,t)\big|_{t=0}=\varphi(\upsilon),\forall\upsilon\in R, & (7)\\ Q(\upsilon,t)\big|_{\upsilon=\upsilon_0}=\psi(t),\forall t\in R_+, & \end{cases}$$

where $(f;Q)$ - the solution of the system (5), (6), $\varphi(\upsilon)=f_0(\upsilon)\exp\left(\int_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)$, (8)

$$\psi(t)=\psi_0(t)\exp\left(\int_{-\infty}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right).$$

Lemma 1. Equation (6) under the conditions (7) and

$$\rho_t'+a(\upsilon)\rho_\upsilon'=0 \qquad (9)$$

is reduced to equivalent form

$$Q(\upsilon,t)=\varphi(\rho(\upsilon,t,0))+\int_0^t\exp\left(\int_{-\infty}^{\rho(\upsilon,t,s)}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty}\kappa(\rho(\upsilon,t,s),\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+$$

$$+\int_0^t\exp\left(\int_{-\infty}^{\rho(\upsilon,t,s)}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F(\rho(\upsilon,t,s),s)ds. \qquad (10)$$

Proof. Putting $\rho\equiv\rho(\upsilon,t,s)$, $\rho_0\equiv\rho(\upsilon,t,0)$, differentiating by t and υ:

$$Q_t'(\upsilon,t)=\varphi'(\rho_0)\rho_{0t}'+\exp\left(\int_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty}k(\upsilon,\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',t))d\upsilon'+$$

$$+\int_0^t\exp\left(\int_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho)}{a(\rho)}\rho_t'\int_{-\infty}^{+\infty}\kappa(\rho,\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+$$

$$+\int_0^t\exp\left(\int_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty}\kappa_\upsilon'(\rho,\upsilon')\rho_t'h(\upsilon')F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+\exp\left(\int_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(t)F(\upsilon,t)+$$

$$+\int_0^t\exp\left(\int_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}\right)\frac{h(\rho)}{a(\rho)}\rho_t'b(s)F(\rho,s)ds+\int_0^t\exp\left(\int_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F_\rho'(\rho,s)\rho_t'ds,$$

$$Q_{\upsilon}^{'}(\upsilon,t)=\varphi'(\rho_0)\rho_{0\upsilon}'+\int\limits_0^t \exp\left(\int\limits_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho)}{a(\rho)}\rho_{\upsilon}'\int\limits_{-\infty}^{+\infty}\kappa(\rho,\upsilon')h(\upsilon')F_0(\upsilon,f(\upsilon',s))d\upsilon'ds+$$

$$+\int\limits_0^t \exp\left(\int\limits_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}\kappa_{\upsilon}^{'}(\rho,\upsilon')\rho_{\upsilon}'h(\upsilon')F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+$$

$$+\int\limits_0^t \exp\left(\int\limits_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho)}{a(\rho)}\rho_{\upsilon}'b(s)F(\rho,s)ds+\int\limits_0^t \exp\left(\int\limits_{-\infty}^{\rho}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F_{\upsilon}'(\rho,s)\rho_{\upsilon}'ds,$$

and placing into (6), we have identity:

$$\exp\left(\int\limits_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}k(\upsilon,\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',t))d\upsilon'+\exp\left(\int\limits_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(t)F(\upsilon,t)\equiv$$

$$\equiv\exp\left(\int\limits_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}k(\upsilon,\upsilon')h(\upsilon')F_0(\upsilon',f(\upsilon',t))d\upsilon'+\exp\left(\int\limits_{-\infty}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(t)F(\upsilon,t),$$

This completes the proof.

As the result we have the equations (5) and (10) – linear algebraical system relative f and Q, where $Q(\upsilon,t)$ may be excluded from system :

$$f(\upsilon,t)=f_0(\rho(\upsilon,t,0))\exp\left(-\int\limits_{\rho(\upsilon,t,0)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)+\int\limits_0^t\exp\left(-\int\limits_{\rho(\upsilon,t,s)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}\kappa(\rho(\upsilon,t,s),\upsilon')h(\upsilon')\times$$
$$\times F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+\int\limits_0^t\exp\left(-\int\limits_{\rho(\upsilon,t,s)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F(\rho(\upsilon,t,s),s)ds\equiv H[f,b],\forall(\upsilon,t)\in\Omega. \quad (11)$$

Lemma 2. Under the conditions (2) and (9) equation (11) is the integral representation of problem (1) - (2).

Proof. By analogy proof lemma 1 differentiating by t and υ and adding $f_t^{'}(\upsilon,t)+a(\upsilon)f_{\upsilon}^{'}(\upsilon,t)+h(\upsilon)f(\upsilon,t)$, we receive the identity $Kf=Kf$.

We note that equation (11) contains two unknown functions $(f(\upsilon,t),b(t))$. Therefore, taking into account condition (3) from (11) we have

$$\psi_0(t)=f_0(\rho(\upsilon_0,t,0))\exp\left(-\int\limits_{\rho(\upsilon_0,t,0)}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)+\int\limits_0^t\exp\left(-\int\limits_{\rho(\upsilon_0,t,s)}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}k(\rho(\upsilon_0,t,s),\upsilon')h(\upsilon')\times$$

$$\times F_0(\upsilon',f(\upsilon',s))d\upsilon'ds+\int\limits_0^t\exp\left(-\int\limits_{\rho(\upsilon_0,t,s)}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F(\rho(\upsilon_0,t,s),s)ds. \quad (12)$$

Further, differentiating by t we obtain

$$\psi_0'(t)=f_{0\upsilon}'(\rho_0)\rho_{0t}'\exp\left(-\int\limits_{\rho_0}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)-f_0(\rho_0)\exp\left(-\int\limits_{\rho_0}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho_0)}{a(\rho_0)}\rho_{0t}'+. \quad (13)$$

$$+\int_{-\infty}^{+\infty} k(\upsilon_0,\upsilon')h(\upsilon')F_0(\upsilon', f(\upsilon',t))d\upsilon' - \int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\times$$

$$\times \rho'_t(\upsilon_0,t,s)\int_{-\infty}^{+\infty} k(\rho(\upsilon_0,t,s),\upsilon')h(\upsilon')F_0(\upsilon', f(\upsilon',s))d\upsilon' ds -$$

$$-\int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty} k'_\upsilon(\rho(\upsilon_0,t,s),\upsilon')\rho'_t(\upsilon_0,t,s)h(\upsilon')F_0(\upsilon', f(\upsilon',s))d\upsilon' ds + \tag{13}$$

$$+b(t)F(\upsilon_0,t) + \int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\rho'_t(\upsilon_0,t,s)b(s)F(\rho(\upsilon_0,t,s),s)ds +$$

$$+\int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F'_\upsilon(\rho(\upsilon_0,t,s),s)\rho'_t(\upsilon_0,t,s)ds.$$

Thus, we have system of equations (11) and (13). Both (11) and (13) if $F(\upsilon_0,t)\neq 0$, $\forall t\in R_+$ are the Volterra type integral equations of the second kind.

Suppose

$$F(\upsilon_0,t)\neq 0,\ \ \forall t\in R_+. \tag{14}$$

Then, into account (11), (13), we have

$$\begin{cases} f(\upsilon,t) = (H[f,b])(\upsilon,t), \\ b(t) = (H_0[f,b])(\upsilon_0,t), \upsilon_0,\upsilon\in R, t\in R_+, \end{cases} \tag{15}$$

where

$$H_0[f,b]\equiv (F(\upsilon_0,t))^{-1}\{\ \psi'_0(t) - f'_{0\upsilon}(\rho_0)\rho'_{0t}\exp\left(-\int_{\rho_0}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right) + f_0(\rho_0)\exp\left(-\int_{\rho_0}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho_0)}{a(\rho_0)}\rho'_{0t} -$$

$$-\int_{-\infty}^{+\infty} k(\upsilon_0,\upsilon')h(\upsilon')F_0(\upsilon', f(\upsilon',t))d\upsilon' + \int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\times$$

$$\times \rho'_t(\upsilon_0,t,s)\int_{-\infty}^{+\infty} k(\rho(\upsilon_0,t,s),\upsilon')h(\upsilon')F_0(\upsilon', f(\upsilon',s))d\upsilon' ds +$$

$$+\int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int_{-\infty}^{+\infty} k'_\upsilon(\rho(\upsilon_0,t,s),\upsilon')\rho'_t(\upsilon_0,t,s)h(\upsilon')F_0(\upsilon', f(\upsilon',s))d\upsilon' ds -$$

$$-\int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\rho'_t(\upsilon_0,t,s)b(s)F(\rho(\upsilon_0,t,s),s)ds -$$

$$-\int_0^t \exp\left(-\int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)b(s)F'_\upsilon(\rho(\upsilon_0,t,s),s)\rho'_t(\upsilon_0,t,s)ds\ \},t\in R_+.$$

If

$$L_H + L_{H_0} = d < 1, \tag{16}$$

where

1) $\sup\limits_{\Omega}\int\limits_0^t \exp\left(-\int\limits_{\rho(\upsilon,t,s)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}k(\rho(\upsilon,t,s),\upsilon')h(\upsilon')\left|F_{0f}(\upsilon',f)\right|d\upsilon' ds \le \gamma_0 = const$,

$$\sup\limits_{\Omega}\int\limits_0^t \exp\left(-\int\limits_{\rho(\upsilon,t,s)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\left|F(\rho(\upsilon,t,s),s)\right|ds \le \gamma_1 = const, L_H = \gamma_0 + \gamma_1 ; \qquad 2)$$

$$\sup\limits_{R_+}\left|(F(\upsilon_0,t))^{-1}\right|\left\{\int\limits_{-\infty}^{+\infty}k(\upsilon_0,\upsilon')h(\upsilon')\left|F_{0f}(\upsilon',f)\right|d\upsilon' + \int\limits_0^t \exp\left(-\int\limits_{\rho(\upsilon_0,t,s)}^{\upsilon}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\left|\rho_t'(\upsilon_0,t,s)\right|\times\right.$$

$$\times\int\limits_{-\infty}^{+\infty}k(\rho(\upsilon_0,t,s),\upsilon')h(\upsilon')\left|F_{0f}(\upsilon',f)\right|d\upsilon' ds + \int\limits_0^t \exp\left(-\int\limits_{\rho(\upsilon_0,t,s)}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\int\limits_{-\infty}^{+\infty}k_\upsilon'(\rho(\upsilon_0,t,s),\upsilon')\left|\rho_t'(\upsilon_0,t,s)\right|\times$$

$$\left.\times\left|F_{0f}(\upsilon',f)\right|h(\upsilon')d\upsilon' ds\right\} \le \gamma_2 = const,$$

$$\sup\limits_{R_+}\left|(F(\upsilon_0,t))^{-1}\right|\left|\int\limits_0^t \exp\left(-\int\limits_{\rho(\upsilon,t,s)}^{\upsilon_0}\frac{h(\upsilon')}{a(\upsilon')}d\upsilon'\right)\left[\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))}\left|\rho_t'(\upsilon_0,t,s)\right|\left|F(\rho(\upsilon_0,t,s),s)\right| + \right.\right.$$

$$\left.\left. + \left|F_\upsilon'(\rho(\upsilon_0,t,s),s)\right|\left|\rho_t'(\upsilon_0,t,s)\right|\right]ds\right| \le \gamma_3 = const, L_{H_0} = \gamma_2 + \gamma_3,$$

then system (15) is solved in $\left(C^{1,1}(\Omega); C(R_+)\right)$, moreover sequences $\{f_{n+1}\}$, $\{b_{n+1}\}$ are constructed by successive approximations method :

$$\begin{cases} f_{n+1} = H[f_n, b_n] \\ b_{n+1} = H_0[f_n, b_n] (n = 0,1,...), \end{cases} \qquad (17)$$

where f_0, b_0 are the initial approximations with errors of calculation:

$$\|f_{n+1} - f\| \le d^{n+1}E_0 \xrightarrow[n\to\infty(d<1)]{} 0,$$

$$\|b_{n+1} - b\| \le d^{n+1}E_0 \xrightarrow[n\to\infty(d<1)]{} 0$$

and

$E_0 = \|f - f_0\| + \|b - b_0\|$.

Thus, we summarize the results as follows.

Theorem 1. Under the conditions (2) – (4), (16) the starting inverse problem is solved in the space setting of functions $\left(C^{1,1}(\Omega); C(R_+)\right)$.

Notice, that the inverse problems are related to ill posed problems and therefore we must get the suitable spaces. If

$$f(\upsilon,t) \in L_h^p(\Omega) \quad , \ b(t) \in L^p(R_+), \ p > 1, \ 0 < \lambda = \lambda(\upsilon,t), \ \frac{1}{p} + \frac{1}{q} = 1, \qquad (18)$$

moreover $\left|(F(\upsilon_0,t))^{-1}\right| \le \gamma_6 = const$, $\int\limits_0^t \left|(F(\upsilon_0,t))^{-1}\right|^p dt \le \gamma_7 = const$, $\forall t \in R_+$,

then by

$$L_0 + L_1 = d_1 < 1, \qquad (19)$$

the solution of inverse problem (1) – (4) may be regarded as limits sequences $\{f_{n+1}\}$, $\{b_{n+1}\}$ in the space $\left(L_h^p; L^p\right)$, where $L_0 = q_2^{\frac{1}{p}}q_4$, $q_2 = \int\limits_{-\infty}^{+\infty}h(\upsilon')d\upsilon'$, $\left|F_{0f}(\upsilon', f(\upsilon',t))\right| \le \lambda(\upsilon',t)$, $\forall(\upsilon',t) \in \Omega$;

$$q_4 = \max(q_1, q_3),\ \ q_1 = \sup_{\Omega} \int_0^t \exp\left(- \int_{\rho(\upsilon,t,s)}^{\upsilon} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) \left(\int_{-\infty}^{+\infty} [k(\rho(\upsilon,t,s),\upsilon')\lambda(\upsilon',s)]^q h(\upsilon') d\upsilon' \right)^{\frac{1}{q}} ds,$$

$$q_3 = \sup_{\Omega} \left(\int_0^t \exp\left(-q \int_{\rho(\upsilon,t,s)}^{\upsilon} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) |F(\rho(\upsilon,t,s),s)|^q ds \right)^{\frac{1}{q}};$$

$$L_1 = \max(\gamma_6\gamma_{1,0};\gamma_6\gamma_{2,0}),\ \sup_{R_+} \left(\int_0^t \left| (F(\upsilon_0,t))^{-1} \right|^p dt \right)^{\frac{1}{p}} = \gamma_6;$$

$$\gamma_{1,0} = \gamma_1 + \gamma_2 + \gamma_3,\ \ \gamma_{2,0} = \gamma_4 + \gamma_5;$$

$$\gamma_1 = \sup_{R_+} \left(\int_{-\infty}^{+\infty} [k(\upsilon_0,\upsilon')\lambda(\upsilon',s)]^q h(\upsilon') d\upsilon' \right)^{\frac{1}{q}},$$

$$\gamma_2 = \sup_{R_+} \left(\int_0^t \exp\left(- \int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) \frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))} |\rho_t'(\upsilon_0,t,s)| \left(\int_{-\infty}^{+\infty} [k(\rho(\upsilon_0,t,s),\upsilon')\lambda(\upsilon',s)]^q h(\upsilon') d\upsilon' \right)^{\frac{1}{q}} ds \right)$$

$$\gamma_3 = \sup_{R_+} \left(\int_0^t \exp\left(- \int_{\rho(\upsilon,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) \left(\int_{-\infty}^{+\infty} [|k_\upsilon'(\rho(\upsilon_0,t,s),\upsilon')||\rho_t'(\upsilon_0,t,s)|\lambda(\upsilon',s)]^q h(\upsilon') d\upsilon' \right)^{\frac{1}{q}} ds \right),$$

$$\gamma_4 = \sup_{R_+} \left(\int_0^t \exp\left(-q \int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) \left(\frac{h(\rho(\upsilon_0,t,s))}{a(\rho(\upsilon_0,t,s))} \right)^q |\rho_t'(\upsilon_0,t,s)|^q |F(\rho(\upsilon_0,t,s),s)|^q ds \right)^{\frac{1}{q}},$$

$$\gamma_5 = \sup_{R_+} \left(\int_0^t \exp\left(-q \int_{\rho(\upsilon_0,t,s)}^{\upsilon_0} \frac{h(\upsilon')}{a(\upsilon')} d\upsilon' \right) |F_\upsilon'(\rho(\upsilon_0,t,s),s)|^q |\rho_t'(\upsilon_0,t,s)|^q ds \right)^{\frac{1}{q}};$$

$\|f\|_{p,h} = \left(\int_{-\infty}^{+\infty} h(\upsilon) |f(\upsilon,t)|^p d\upsilon \right)^{\frac{1}{p}}$ is limited by t, $\|b(t)\|_p = \left(\int_0^t |b(s)|^p ds \right)^{\frac{1}{p}}$.

Really, valuing in L_h^p and L^p and taking into consideration (17) , $|f_{n+1} - f|$, $|b_{n+1} - b|$, we have

$$\|f_{n+1} - f\|_{p,h} \le L_0 (\|f_n - f\|_{p,h} + \|b_n - b\|_p),$$

$$\|b_{n+1} - b\|_p \le L_1 (\|f_n - f\|_{p,h} + \|b_n - b\|_p).$$

So taking account (19) obtain

$$\begin{aligned} &\|f_{n+1} - f\|_{p,h} + \|b_n - b\|_p \le d(\|f_n - f\|_{p,h} + \|b_n - b\|_p) \le \ldots \le d^{n+1}(\|f - f_0\|_{p,h} + \|b - b_0\|_p) \le \\ &\le d^{n+1} E_* \xrightarrow[n\to\infty(d<1)]{} 0, \end{aligned} \tag{20}$$

in $(L_h^p; L^p)$ sense

$$E_* = \|f - f_0\|_{p,h} + \|b - b_0\|_p.$$

A limitation of functions $f(\upsilon,t)$, $b(t)$ in our space setting are obviously (proof is made on base (15) taking account that free parts of system (15) are limited in (L_h^p, L^p)).

We summarize as follows.

Theorem 2. If (18), (19) are fulfilled, then inverse problem (1) – (4) has the unique solution in $W_p = (L_h^p(\Omega); L^p(R_+))$, moreover this solution is regarded as the limits sequences $\{f_{n+1}\}$, $\{b_{n+1}\}$ in W_p space setting.

Discussion

This paper is close to conception [7]. In [7] were studied the direct problems for the differential and integro-differential equations. The above-investigated integral conversion may be used for the linear and non-linear integro-differential and differential equations, for the direct and inverse problems. The solutions are regarded in integral forms moreover in the direct problems for the differential equations they are regarded in obvious forms.

One may move on to use this integral forms in numerical calculations for the concrete applied sciences. Subsequently it would be interesting to consider the case non-linear coefficient $a(\upsilon, f(\upsilon,t))$ and the case

$$\int_{-\infty}^{+\infty} h(\upsilon)d\upsilon = +\infty .$$

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